ABSTRACT: The assessment of rock mass strength is a key element for the analysis of many rock excavations, both open pit and underground. There have been attempts to assess this strength using a suitable rock mass strength criterion having input derived using a Rock Mass Classification system. The most well developed criterion is the Hoek-Brown Failure Criteria using the Geotechnical Strength Index (Hoek, 2002) as input. Despite its popularity, the GSI has some inconsistencies. Rock masses with differing fracture frequencies and joint conditions can have the same GSI value and therefore the same strength characteristics would be applied. To address this issue, this paper describes a method to determine the input to the criterion on the basis of the joint condition and fracturing of the rock mass.

1 INTRODUCTION

The assessment of rock mass strength is a key element in any rock excavation for both, open pit and underground excavations. When numerical models are used as a tool of analysis, this strength is defined in terms of a strength envelope. The envelope may be linear, like Mohr – Coulomb or non linear like that suggested by Hoek [1995].

There are empirical methods that relate rock mass classifications with design parameters like slope angles, open-spans and support, Bieniawski [1989], Laubscher [1990], Barton [1974]. The classification methods do not however directly provide the strength characteristics.

The non-linear Hoek – Brown criterion relates the strength envelope to the rock mass classification through the GSI index. This method allows strength assessment based on actual data collected on site, mainly discontinuity density, discontinuity condition, plus laboratory information like unconfined compressive strength.

One drawback of this method is that different characteristics of the rock mass, like joint condition and discontinuity density, are combined in a single number, GSI, and this number is used to assess the rock mass strength. Two different rock masses can have the same GSI (Figure 1) but if the joint condition and discontinuity density are different, it is unlikely that both rock masses would have the same strength. If the joint condition and discontinuity density are considered separately it is possible to estimate the rock mass strength for each one.

2 ROCK MASS STRENGTH ASSESSMENT BASED ON ROCK MASS RATING GSI

The rock mass strength can be evaluated using the Hoek – Brown failure criteria [Hoek, 1995]. The strength envelope written in term of the principal stresses is shown in Eq. (1).

$$\frac{\sigma_1}{\sigma_{ci}} = \frac{\sigma_3}{\sigma_{ci}} + \left( m_b \left( \frac{\sigma_3}{\sigma_{ci}} + s \right) \right)^a$$

(1)

Where $\sigma_{ci}$ is the unconfined compressive strength of the intact rock (UCS), and the parameters $m_b$, $s$ and $a$ are related with the rock mass rating through the GSI. These relationships are shown in Eq (2), (3) and (4).
Figure 1: Correlation between GSI and rock mass properties and $N_f$ and $N_c$ parameters definition. Rocscience [8]

\[
\frac{m_i}{m_i} = \exp \left( \frac{GSI - 100}{28 - 14D} \right) \tag{2}
\]

\[
s = \exp \left( \frac{GSI - 100}{9 - 3D} \right) \tag{3}
\]

\[
a = \frac{1}{2} + \frac{1}{6} (e^{-GSI/15} - e^{-20/3}) \tag{4}
\]

where $m_i$ is a material constant and $D$ is a Disturbance Factor.

A correlation between the GSI index and the rock mass quality can be seen in Figure 1.

3 ROCK MASS STRENGTH ASSESSMENT BASED ON JOINT SPACING AND JOINT CONDITIONS

The GSI is insensitive to those rock mass characteristics that most influence rock mass strength. A series of relationships are proposed that seek to partially address this issue by differentiating rock masses having the same GSI. These relationships are functions of two parameters, $N_f$ and $N_c$. $N_f$ is a function of the characteristics of the fracturing and interlocking and $N_c$ is a function of the joint condition. The variation of these parameters is conceptually shown in Figure 1. For example a blocky rock with fair joint surface condition might have a GSI=35, in this case the blocky conditions is characterized by the parameter $N_f$ and the fair surface condition by the parameter $N_c$. 
The recommended strength envelope in Eq. (1), uses the same $m_i$ parameter as currently used in the Hoek – Brown failure criterion however it is proposed that $m_b$, $s$ and $a$ are calculated using the following relationships.

\[
\frac{m_b}{m_i} = \frac{1}{N_{c_{\text{max}}}} \left[ N_c \left( f_1(N_f) - f_2(N_f) \right) + N_{c_{\text{max}}} f_2(N_f) \right] 
\]

(5)

\[
s = \frac{1}{N_{c_{\text{max}}}} \left[ N_c \left( f_3(N_f) - f_4(N_f) \right) + N_{c_{\text{max}}} f_4(N_f) \right] 
\]

(6)

\[
a = 0.6 + 0.4 \left[ \exp \left( -N_f \times 0.15 \right) - \exp \left( -40 \times 0.15 \right) \right] 
\]

(7)

The functions $f_i(N_f)$ and the parameter $N_{c_{\text{max}}}$ are functions of the Rock Mass Classification system used. The functions $f_i(N_f)$ used in Eq. (5) and (6) can be written using the generic form shown in Eq. (8)

\[
f_i(N_f) = A + (B - A) \left[ \exp(-C(M - N_f)) - \exp(-CM) \right]
\]

(8)

The parameters $A$, $B$, $C$ and $M$ used on each function $f_i(N_f)$ are shown in Table 1.

Table 1: Parameters $A,B,C$ and $M$

<table>
<thead>
<tr>
<th>Function</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.14</td>
<td>0.69</td>
<td>0.25</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>0.04</td>
<td>0.60</td>
<td>0.25</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>4e-3</td>
<td>0.25</td>
<td>0.6</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
<td>4e-4</td>
<td>0.6</td>
<td>40</td>
</tr>
</tbody>
</table>

It is recommended that the standard rock mass classification systems are used to evaluate the parameters $N_f$ and $N_c$. It is not therefore necessary to obtain any additional data to that currently required as input to these systems. The ratings for spacing of discontinuities and joint condition should be used to evaluate the parameters $N_f$ and $N_c$. It is important to note this paper do not recommend to use GSI directly to evaluate the rock mass strength.

In this paper, two systems are recommended to be used with the Eq. (5), (6) and (7), Bieniawski [1989], and Laubscher [1990].
The parameters \( m_b/m_i \) and \( s \) are related to changes in fracture frequency and joint condition. Hoek [1995] presented the indicative values for these parameters shown in Figure 2. The values of \( A \), \( B \) and \( C \) in Table 1 were based on the values \( m_b/m_i \) shown in this figure.

**Function 1** describes the changes in \( m_b/m_i \) with changes in the fracture frequency for VERY GOOD surface condition. The maximum and minimum values for \( m_b/m_i \) are 0.60 and 0.17 shown in Fig. 2. These values were extrapolated to 0.69 and 0.14, considering Fig 1 has 6 degrees of fracturing and Fig. 2 shows only 4.

**Function 2** describes the changes in \( m_b/m_i \) with changes in the fracturing for VERY POOR surface condition. The maximum and minimum values for \( m_b/m_i \) are 0.08 and 0.04. The value of 0.08 was increased to 0.60 because if the number of joints is small the joint condition become less important because there is a few or no joint in any given block of rock mass. For this reason the ratio \( m_b/m_i \) must increase and become slightly lower than the ratio \( m_b/m_i \) for massive rock with a few joints with good surface condition. The minimum value of 0.04 was kept the same.

**Function 3** describes the changes that occur to the value of \( s \) with changes in the fracture frequency for VERY GOOD surface conditions. The maximum and minimum values for \( s \) are 0.19 and 0.17 shown in Fig. 2. The maximum value was increased to 0.25 to consider those rock masses having very few joints as shown in Figure 1 for massive rock and very good surface conditions. The minimum value of 0.04 was kept the same.

**Function 4** describes the changes in \( s \) with changes in the fracturing for VERY POOR surface condition. The maximum and minimum values for \( s \) are 0.0004 and 0.0 as indicated in Figure 2. These values where kept without modification.
Parameter $a$. In the Hoek – Brown failure criterion the parameter $a$ varies between 0.5 to 0.66 for intact rock to poor quality rock. In this paper it is proposed to use Eq. (7) that allows the parameter to vary between 0.6 to 1.0 with changes in $N_f$. Those numbers were based on the non linear strength envelope of highly fractured rock like rockfill that has an $a$ parameter between 0.63 to 0.89 Indraratna [1993].

3.1 ROCK MASS STRENGTH BASED ON BIE NIAWSKI’S ROCK MASS RATING SYSTEM RMR$_B$

The basic parameters used in Bieniawski’s Rock Mass Rating ($RMR_B$) system are:

- Strength of Intact Rock $\sigma_{ci}$
- Drill Core Quality $RQD$
- Spacing of discontinuity
- Condition of Discontinuities (based on discontinuity length, aperture width, roughness, infill type and thickness of the weathering)
- Ground Water

If $RMR_B$ is used as a Rock Mass Classification system, the $N_f$ and $N_c$ parameters must be calculated using Eq. (9) and (10)

\[ N_f = \left( RQD_{\text{rating}} + \text{Spacing}_{\text{rating}} - 8 \right) \frac{40}{32} \]  \hspace{1cm} (9)

\[ N_c = \text{Discontinuity} \_ \text{Condition}_{\text{rating}} \]  \hspace{1cm} (10)

$N_f$ would be in the range of 0 to 40 points and $N_c$ in the range of 0 to 30. $N_{c_{\text{max}}} = 30$ for Bieniawski’s system.

3.2 ROCK MASS STRENGTH BASED ON LAUBSCHER’S ROCK MASS RATING SYSTEM RMR$_L$

The basic parameters used in Laubscher’s [1990] Rock Mass Rating ($RMR_L$) system are:

- Strength of Intact Rock $\sigma_{ci}$
- Drill Core Quality $RQD$
- Space Discontinuity
- Fracture Frequency
- Condition of Discontinuity (Joint Condition)

The system provides two ways to assess the fracturing. $RQD$ and Space Discontinuity can be used together or, instead, Fracture Frequency can be used. In both cases the rating varies from 0 to 40.

For joint condition the rating varies from 0 to 40.

The parameter $N_f$ can be calculated using Eq. (11) or (12), the parameter $N_c$ is calculated using Eq. (13).

\[ N_f = RQD_{\text{rating}} + \text{Spacing}_{\text{rating}} \]  \hspace{1cm} (11)

\[ N_f = \text{Fracture} \_ \text{Frequency}_{\text{rating}} \]  \hspace{1cm} (12)

\[ N_c = \text{Discontinuity} \_ \text{Condition}_{\text{rating}} \]  \hspace{1cm} (13)

$N_f$ could be in the range of 0 to 40 and $N_c$ could be in the range of 1 to 40.
$N_{c_{\text{max}}} = 40$ for Laubscher’s system.

4 TENSILE STRENGTH AND EQUIVALENT MOHR – COULOMB STRENGTH ENVELOPE

The tensile strength $\sigma_t$ for the rock mass can be calculated by setting $\sigma_1=\sigma_2=\sigma_t$ in Eq. (1) and can be calculated using Eq. (14). This represents a biaxial tension. For brittle materials, the uniaxial tensile strength is equal to the biaxial tensile strength (Hoek, 1983).

$$\sigma_t = -\frac{s\sigma_{\text{ci}}}{m_b}$$

(14)

It is possible to calculate an equivalent Mohr – Coulomb strength envelope for a range of confinement defined by $\sigma_t<\sigma_1<\sigma_{3_{\text{max}}}$ (Hoek, 2002). The following equations can be used to assess the equivalent friction $\phi$ and cohesion $c$.

$$\phi = \sin^{-1}\left[\frac{6am_b(s + m_b\sigma_{3n})^{a-1}}{2(1+a)(2+a) + 6am_b(s + m_b\sigma_{3n})^{a-1}}\right]$$

(15)

$$c = \frac{\sigma_{\text{ci}}[(1+2a)s+(1-a)m_b\sigma_{3n}](s + m_b\sigma_{3n})^{a-1}}{(1+a)(2+a)(1+(6am_b(s + m_b\sigma_{3n})^{a-1}/((1+a)(2+a)))^{-1}}$$

(16)

Where $\sigma_{3n} = \sigma_{3_{\text{max}}}/\sigma_{\text{ci}}$

An alternative assessment for the tensile strength can be derived replacing $\sigma_1=0$ and $\sigma_3=\sigma_t$ in equation 1. This gives us equation 17.

$$-\sigma_t = \sigma_c \left( m_b \frac{\sigma_t}{\sigma_c} + s \right)^a$$

(17)

Equation 17 can be solved using iterative methods. It is proposed to use equation 18 with a starting value for $\sigma_t^1$, given by equation 19 where $\sigma_t$ is calculated with equation 14. In general this method produce tensile strength lower than equation 14.

$$\sigma_t^{i+1} = -\left(\frac{\sigma_t^i}{\sigma_c}\right)^{\frac{1}{a}} - s \left(\frac{\sigma_c}{m_b}\right)$$

(18)

$$\sigma_t^1 = \frac{\sigma_c\sigma_t}{\sigma_c - \sigma_t}$$

(19)
5 EXAMPLES

5.1 STRENGTH ASSESSMENT BASED ON BIENIAWSKI’S ROCK MASS RATING SYSTEM RMRB.

As an example of the application of the proposed system, two rock masses with the same characteristics except joint condition and fracturing are considered. The change in the ratings is such that both have the similar $RMR_B=62$ with $\sigma_c=100$ MPa and $m_i=12$.

Table 2 shows the ratings for the different rock mass characteristics and the values $N_f$ and $N_c$ used in the strength calculation. Includes the Hoek – Brown strength parameters and the equivalent Mohr – Coulomb strength parameters for a $\sigma_{3\text{max}}=2$ MPa.

Table 2: Strength Parameters Based on Bieniawski’s rock mass rating system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case A Rating</th>
<th>Case B Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS of intact rock</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>RQD</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>Spacing</td>
<td>17</td>
<td>4</td>
</tr>
<tr>
<td>Joint Condition</td>
<td>5</td>
<td>26</td>
</tr>
<tr>
<td>Water</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$RMR_B$</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>$m_b$</td>
<td>1.58</td>
<td>1.52</td>
</tr>
<tr>
<td>$s$</td>
<td>1.01e-3</td>
<td>3.47e-3</td>
</tr>
<tr>
<td>$a$</td>
<td>0.602</td>
<td>0.762</td>
</tr>
<tr>
<td>$\phi$</td>
<td>47º</td>
<td>38º</td>
</tr>
<tr>
<td>$c$ [MPa]</td>
<td>0.55</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Case A is a rock with less fracture than rock B but with soft gauge in the joints. Case B is a rock with high level of fracturing with slightly rough surface and weathered surfaces. Because the number of fractures is less in Case B compared with Case A and the joint conditions is different the strength calculated is different despite having both the same $RMR$. Figure 3 shows the strength envelope for both cases.
5.2 STRENGTH ASSESSMENT BASED ON LAUBSCHER’S ROCK MASS RATING SYSTEM RMR_L.

The following example estimates the strength of a highly fractured rock mass having clean rough joints.

The ratings applicable to the different characteristics are shown in Table 3. The basic parameters are: $\sigma_i = 75$ MPa and $m_i = 12$. The rock mass classification based on Laubscher’s [1990] and Bieniawski’s [1989] systems are $RMR_L = 50$ and $RMR_B = 60$ ($GSI = 55$).

The parameters listed in the table are those applicable to Bieniawski’s system. These data were used to determine the $GSI$. The strength of the rock mass was determined using the Hoek – Brown criterion as applicable to disturbed and undisturbed rock. The equivalent Mohr – Coulomb parameters (i.e, friction $\phi$ and cohesion $c$,) calculated for $\sigma_{\text{max}} = 2$ [MPa] are included in the table ($\sigma_{\text{max}}$ was chosen arbitrarily in this example for further discussion see reference [8]).

Table 3: Comparison Strength Using Proposed Method and Hoek – Brown Failure Criteria

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Laubscher RMR ratings</th>
<th>Bieniawski RMR ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCS of intact rock</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>RQD (0%)</td>
<td>0</td>
<td>$N_f = 2$</td>
</tr>
<tr>
<td>Spacing (mm)</td>
<td>2</td>
<td>$N_c = 40$</td>
</tr>
<tr>
<td>Joint Condition</td>
<td>40</td>
<td>$N_j = 30$</td>
</tr>
<tr>
<td>Water</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>RMR_B</td>
<td>50</td>
<td>60 ($GSI = 55$)</td>
</tr>
<tr>
<td>Disturbance</td>
<td>$D = 0$</td>
<td>$D = 1$</td>
</tr>
<tr>
<td>$\phi$ (°)</td>
<td>34°</td>
<td>51°</td>
</tr>
<tr>
<td>$c$ [MPa]</td>
<td>0.165</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.61</td>
</tr>
</tbody>
</table>
Table 3 shows the method proposed to assess the rock mass strength gives reasonable values of Mohr – Coulomb strength parameters. Using the GSI index the strength of a highly fractured clean rock is extremely high for both, disturbed \((D=1)\) and undisturbed rock \((D=0)\). A material with cohesion in the range of 0.61 to 1.17 MPa can stand vertical between 90 to 170m without considering the strength due to friction. It is unlikely a highly fractured rock can sustain a vertical slope with these heights.

6 CONCLUSION

A new approach is presented to assess the parameters \(m_b\), \(s\) and \(a\) in the Hoek – Brown failure criterion. The method uses the joint condition and fracture frequency via RQD and spacing as input to the criterion.

Two Rock Mass Classification systems are considered; Bieniawski’s and Laubscher’s. Instead of using the overall Rock Mass Rating RMR derived using one of these systems as input to the strength criterion, two new parameters based on joint condition and fracturing of the rock mass \(N_f\) and \(N_c\) are used. These new parameters can be evaluated directly from the RMR and no new data is required.

The procedure can address differences in joint condition and fracturing that are not reflected in the rating systems. The proposed method can differentiate between the strength of rock masses with high fracturing and clean rough joints and rock masses with low fracturing and smooth joints with gouge, even though they might have the same rock mass ratings.

REFERENCES