Improved Grade Estimation using Optimal Dynamic Anisotropy Resolution

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Grade estimation based on kriging uses a search ellipsoid centred on each block to select samples used for estimation. Traditionally, a globally orientated search ellipsoid is used during the estimation process. In folded structures or meandering channels where the orientation of grade continuity changes, misalignment of the search ellipsoid by just a few degrees can impact the estimation results. The estimation results can be improved if the search ellipsoid is aligned alongside the direction and continuity of the mineralisation. The use of the Dynamic Anisotropy option during the estimation process allows the anisotropic rotation angles defining the search ellipsoid to locally honour the trend of the mineralisation within each cell of the block model. This paper describes the application of Dynamic Anisotropy to a slightly undulating area which lies on a gently folded limb of a syncline at Driefontein gold mine in South Africa. Estimates calculated using Dynamic Anisotropy and the traditional method (Ordinary Kriging) were compared on a real-life dataset. The results of the study showed that generally, the application of the Dynamic Anisotropy interpolation technique during the estimation process slightly improves the quality of estimates. By comparison with the Dynamic Anisotropy technique, Ordinary Kriging is a better method of grade estimation with increasing block size.

Introduction

The results of a grade estimation exercise can be improved if the continuity in undulating, meandering or folded geological structures can be matched by aligning the search ellipsoid along the direction and continuity of the mineralisation trend in the ore body. The method referred to as Dynamic Anisotropy (DA) (CAE Mining), allows the anisotropic rotation angles defining the search ellipsoid and variogram model, to locally honour the trend of the mineralisation for each cell within a block model. This paper confirms that the application of DA to a slightly undulating, gently folded limb of a syncline in an area of the Driefontein gold mine provides a slight improvement to the grade estimation process. In addition, the optimal DA resolution providing this improvement in grade estimates is determined. Currently, the mine uses Ordinary Kriging (OK) method of estimation on a 30×30 m two-dimensional (2D) block model to calculate their Mineral Resource estimates.

Background

The South African gold mining industry utilizes geostatistical techniques extensively to provide estimates for gold reserves and resources, and to assist with daily mine valuation and mine planning (Krige 2000). Geostatistical estimation is used in the mining industry to provide best linear unbiased estimates of metal accumulation grades in unmined areas (Armstrong, 1998). The reliability of Mineral Resource estimates depends not only on the quality and number of samples, but also on the degree of continuity of the mineralisation (Blais and Carlier, 1968). The more continuous the mineralisation, the better will be the correlation between the grade of a sample and the unknown true grade of the block represented by this sample. In the grade estimation process, grades are interpolated or extrapolated into 2D or three-dimensional (3D) block models of a deposit. The process uses a search volume ellipsoid (Figure 1) centred on each block, to select samples for use in one of the several interpolation algorithms to estimate the block grade.

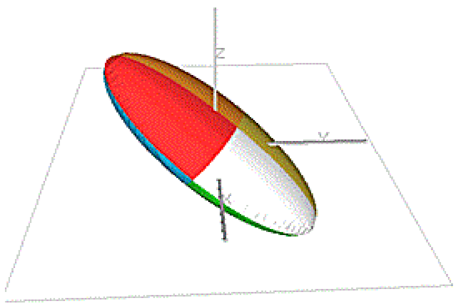

Figure 1: Oriented search ellipsoid (Source: Copyright © Datamine 2016).

In anisotropic geological settings, the continuity is a function of the mineralised structure or lithology and the long axis of the search ellipsoid should be oriented along the most continuous direction of the mineralisation, as shown in Figure 3.

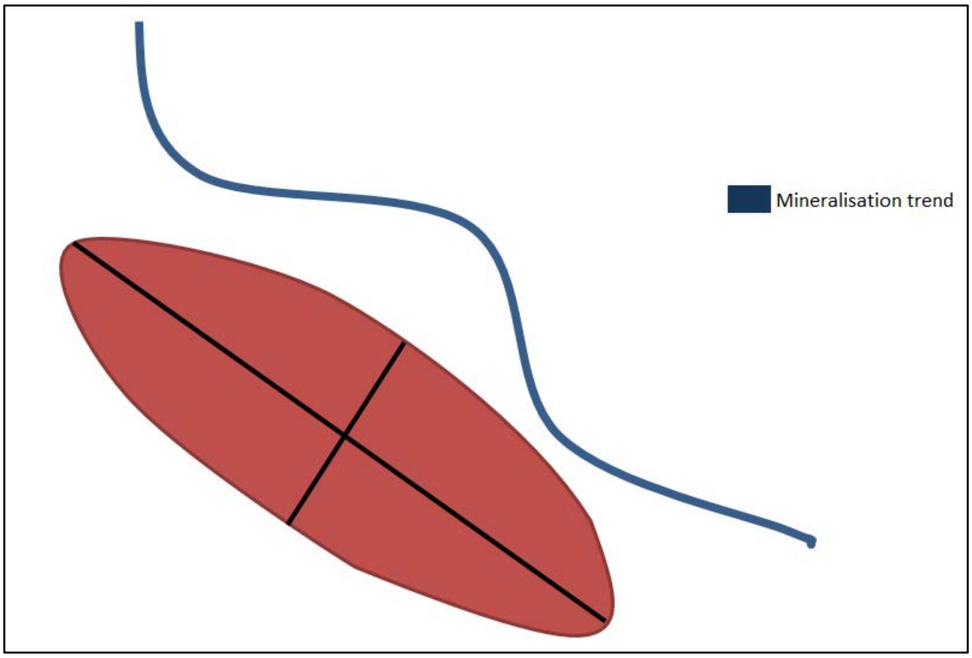

Figure 2: Search ellipsoid oriented along most continuous direction of mineralisation.

Misalignment of the search ellipsoid by just a few degrees can affect the estimation results (CAE Mining) e.g. a block might be estimated to be below the cut-off grade when it is in fact above the cut-off grade and vice-versa. The size, shape and orientation of the search ellipsoid are determined from variography analyses identifying the major directions of continuity of mineralisation in the deposit (Zabrusky, 2013). Spatial variability of a regionalised variable maybe isotropic: if the axes (X, Y and Z) of the ellipsoid are of equal length, or anisotropic if the axes are of different lengths in orthogonal directions. The use of an anisotropic search window produces a small but consistent improvement in all estimating techniques for all geological settings (Isaaks and Srivastava, 1989).

Grades in gold vein deposits are characterised by a highly positively skewed distribution in which a small number of higher grade samples can cause an overestimation in the surrounding low grade estimation block areas (Kim *et al*., 1987; Dominy *et al*., 1997). These outliers can also cause variogram distortions such as pure nugget effect, which in turn affects the grade estimate results (Roy, 2000).

Variogram parameters are used to create the search volume ellipsoid and during DA estimation, the ellipsoids is centred on each block, to select samples used for estimation and is oriented to follow the mineralisation trend precisely as shown Figure 3.

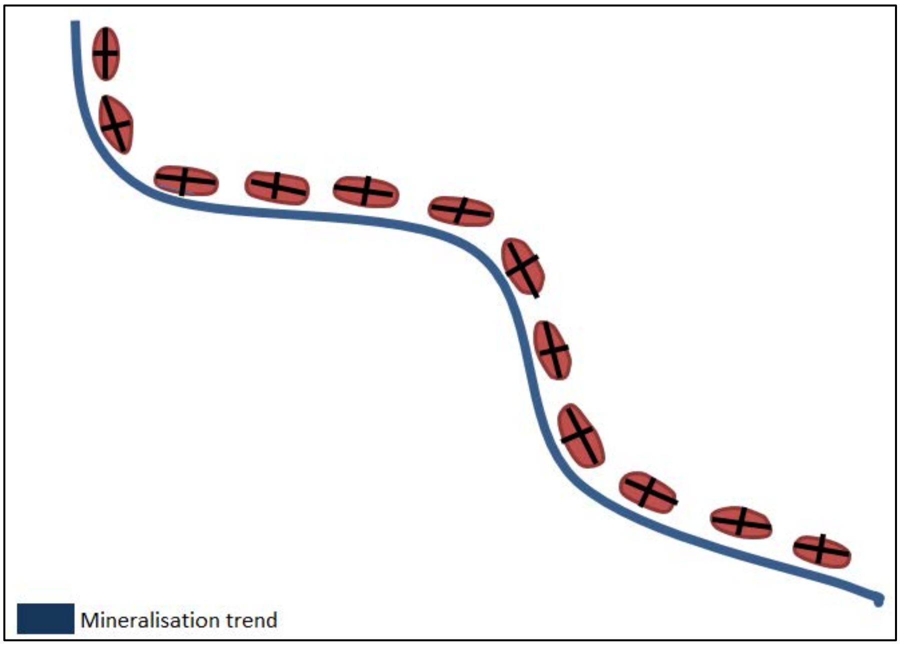

Figure 3: Ellipsoids oriented to follow mineralisation trend precisely.

Case Study

Sibanye currently owns and operates four underground and surface gold operations, namely Driefontein, Kloof and Cooke in the West Witwatersrand region and the Beatrix in the southern Free State, as shown in the locality map of Figure 2. The Driefontein mine where this research was undertaken lies 70 to 80 km west of Johannesburg in South Africa.

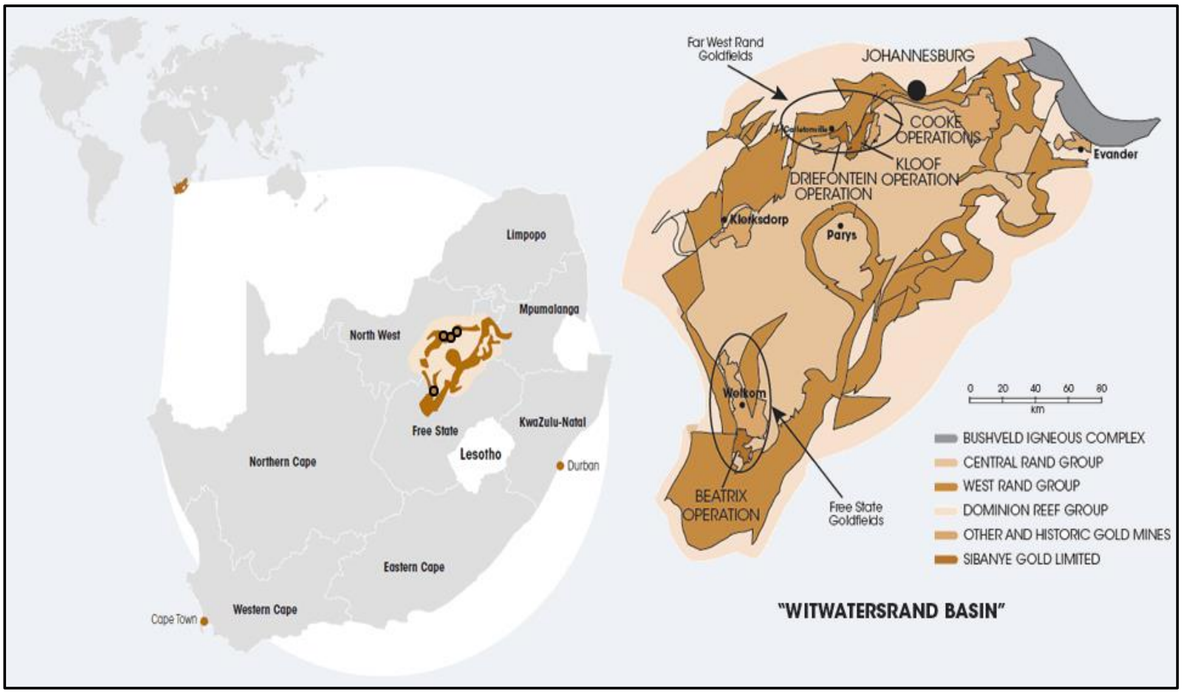

Figure 4: Sibanye location map (Source: https://www.sibanyegold.co.za/operations, 2015).

Geological Model and Interpretation

The varying support of the intersections of mineralised rock in gold deposits with narrow vein- or layer-like morphology (Bertoli *et al*., 2003) means that the variable of interest (i.e. grade) is not a suitable variable for direct kriging. Although the width of the reef may vary from a few centimetres to more than two meters on any one Witwatersrand-type gold mine, the entire seam is mined and estimation of the grade is therefore executed in 2D thus ensuring that bias due to non-additivity of grade is avoided. In the 2D approach, the product of the channel width (cm) and gold grade (g/t), known as the accumulation value (cmg/t) is also amenable to direct kriging (Chiles and Delfiner, 2012). Hence accumulation is kriged independently of channel width and the gold grade is derived by dividing the two estimates by one another.

Driefontein geological models are based on structural, grade and sedimentological data. Sedimentological, gold value and channel width (CW) data are used to delineate the ore body into locally and geologically homogeneous zones called geozones. The boundaries of the geozones constrain the 2D geostatistical estimation of the Mineral Resources based on surface and underground boreholes and channel samples.

Statistical Analysis

Descriptive statistics of the entire dataset are presented in Table 1 and are shown in the histogram and Q-Q plots of Figure 5.

Table 1: Statistics from the sample file

Field	NSamp	Min	Max	Mean	Stddev	Skewness	Kurtosis	CoV
cm.g/t	46 943	0.00	14 250.90	2 218.85	2 250.84	2.00	4.77	1.01
CW/cm	46 943	1.00	0.62	26.07	18.64	3.62	23.81	0.71

The mean value for cm.g/t and CW is 2218.85 cm.g/t and 26.07 cm respectively and the coefficient of variation (CoV) values are 1.01 and 0.71 for cm.g/t and CW, respectively. These values are within acceptable ranges. This indicates that there are not a significant number of extreme values. The samples are closely spaced which translates to a massive quantity of values informing the distribution very well

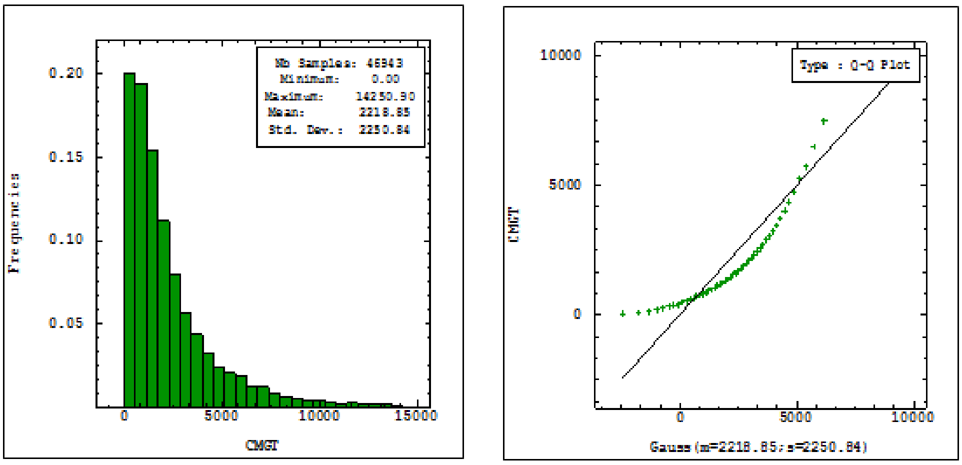

Figure 5: Histogram and Q-Q plot of accumulation values (cmg/t)

As expected, the histogram and Q-Q plot show that the gold grade distribution is leptokurtic and positively skewed. The points on the Q-Q plot depart from a straight line and the distribution is skewed to the right.

Data Declustering

Sampling programs in ore bodies are not entirely random and more often than not, mineralised rocks are unevenly sampled leading to clustering of sample points, particularly in the high grade areas. Preferential sampling of spatially auto-correlated data could mean that the histogram of the sample data may not reflect the histogram of the population. The declustering method assigns higher weights to the less densely sampled points and compensates for redundancy in densely sampled areas. The process adjusts the summary statistics to be representative of the entire area of interest thereby removing potential bias (Deutsch and Journel, 1998). The dataset used in this study was declustered and normal score transformed using Gaussian anamorphosis using hermite polynomials in order to satisfy the assumptions of continuous variability was used.

Gaussian Anamorphosis

Gaussian anamorphosis (normal scores) transforms the raw variable, cm.g/t into a Gaussian variable using the empirical inversion method. For each raw value, the method calculates the attached empirical frequency and the corresponding Gaussian value; this function can be conveniently written as a polynomial expansion:

$$\varphi(Y) = \sum_{i=0}^{\infty} \Psi_i H_i\,(Y)$$

where $H_i(Y)$ are called the Hermite Polynomials. In practice, this polynomial expansion is stopped at a given order. Using this technique, any function can be expressed in terms of Hermite polynomials (30 polynomials were used in this study), (Baafi and Schofield, 1997). Figure 6 shows the anamorphosis model which is represented with dotted lines when outside the validity bounds called the practical interval of definition of the raw variable. These bounds are the points where the model is no longer smoothly increasing.

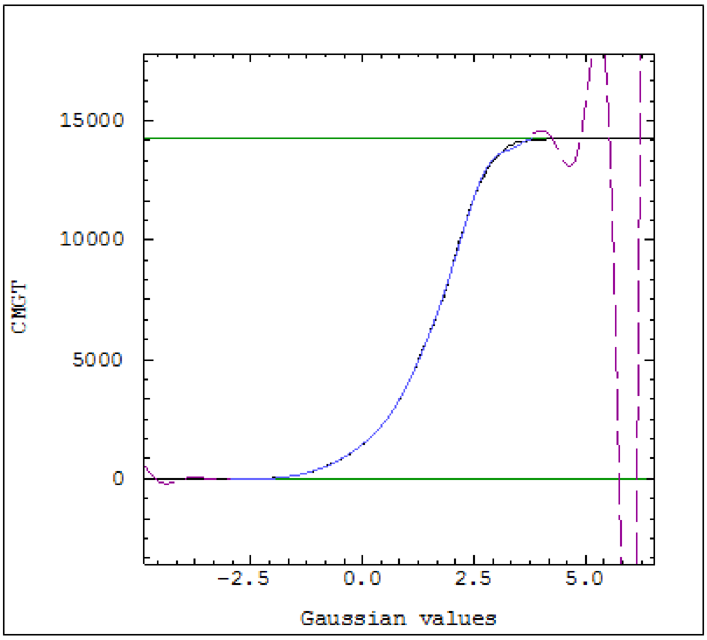

Figure 6: Anamorphosis plot – cm.g/t

The absolute interval of definition is represented as two horizontal lines. The solid blue line represents the cumulative solution of the hermite polynomials, which informs the Gaussian Anamorphosis, and the black line represents the actual data. A histogram and Q-Q plots for the transformed accumulation values are shown in Figure 7.

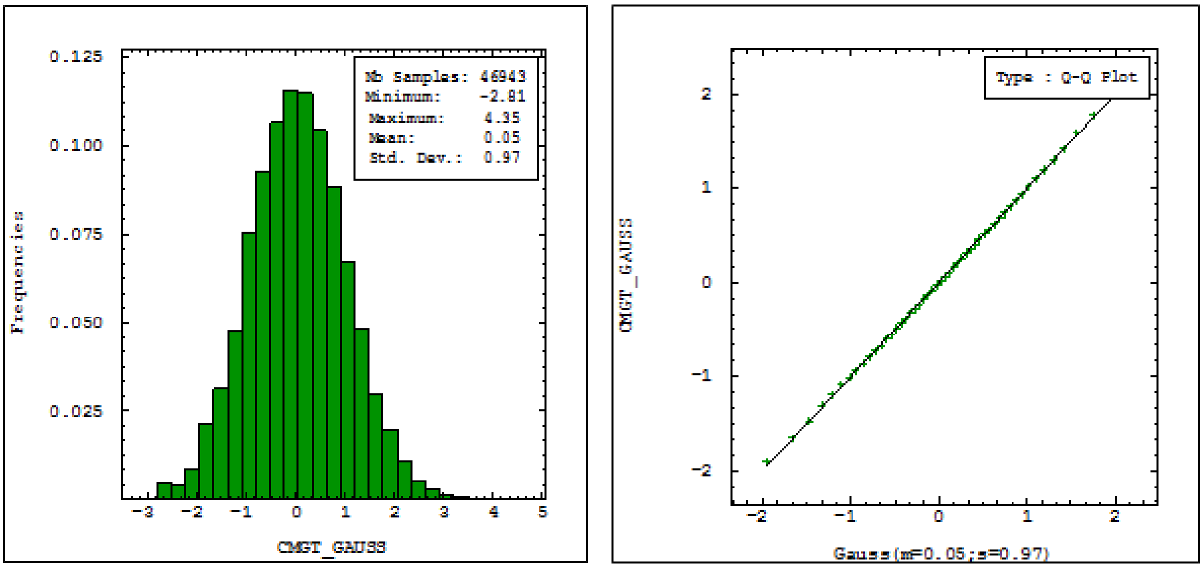

Figure 7: Gaussian Cm.g/t histogram and Q-Q plots.

It can be shown from Figure 7 that the histogram plot shows a normal distribution and the Q-Q plot follows an almost linear trend thus confirming that a normal scores model is a good approximation for the distribution.

Calculating Semi-Variogram

The omni-directional semi-variogram shown in Figure 6 was modelled using normal scores cmg/t data in Geovariances Isatis software, to produce a spherical model of the form:

$$\gamma(h) = \begin{cases} c_0 \left[\frac{3}{2}\frac{h}{a_0} - \frac{1}{2}\left(\frac{h}{a_0}\right)^3\right], & for\ h \leq a_0 \\ c_0, & for\ \ h > a_0 \end{cases}$$

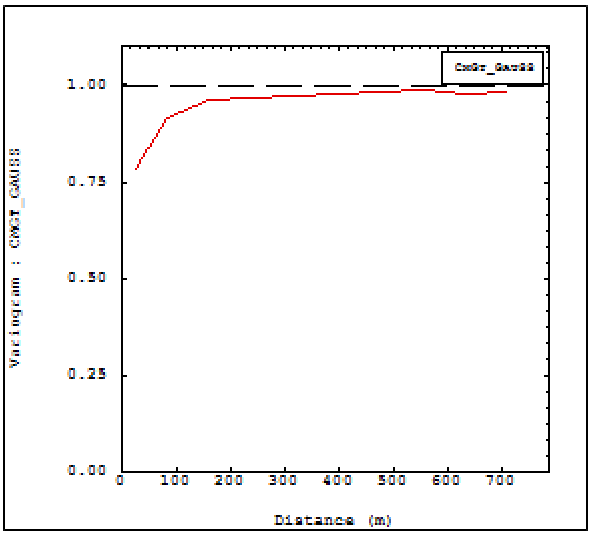

Figure 8: Omni – directional experimental variogram model for cm.g/t

The standardised variogram model shown in Figure 8 has a range of 180 m and a very high nugget effect (approximately 75 % nugget to sill ratio) which is expected for a gold deposit. However, Ordinary Kriging with a very high nugget effect may produce worse estimates than Inverse Distance methods and is most successful when anisotropy is properly described and when the semi-variogram is locally customised (Isaaks and Srivastava, 1989). The variogram map shown in Figure 9 was constructed to identify directions in which the data has the greatest continuity.

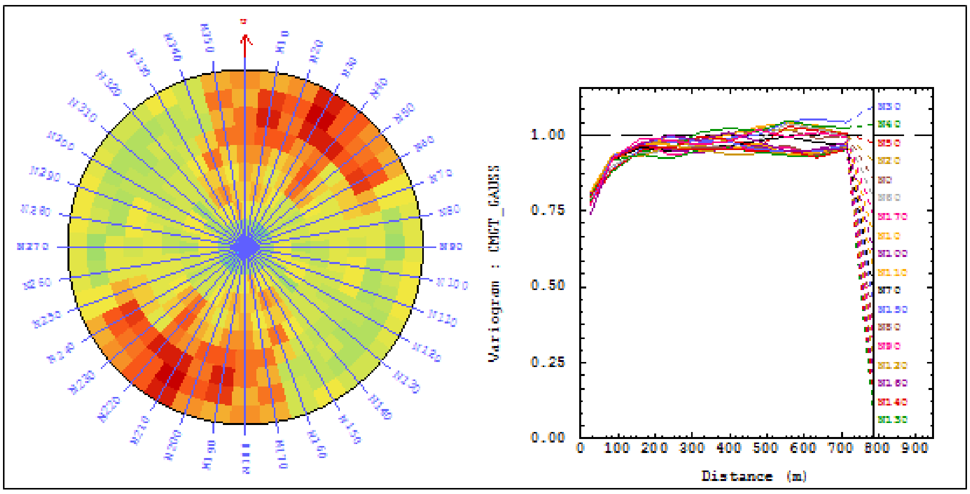

Figure 9: Variogram map plot for normal scores of cm.g/t data

The variogram map indicates the greatest continuity occurs along a northwest-southeast direction with an orthogonal northeast-southwest direction for the least continuity; N140° and N50° were used in the model variogram. Figure 10 shows the variogram models for Gaussian transformed cm.g/t data in the short (N50°, left) and long (N140°, right) directions of anisotropy, respectively.

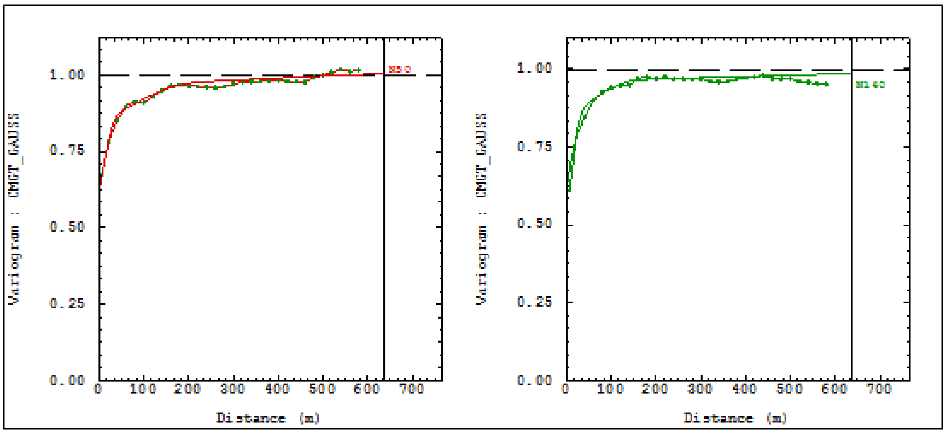

Figure 10: Variogram models for short (N50, left) and long (N140, right) directions

The model variogram in Figure 10 was back-transformed from Gaussian cm.g/t into a discretized variogram Figure 11 using the already saved anamorphosis function (transformation table) calculated during the transformation of the raw cm.g/t into the Gaussian cm.g/t. The raw variable (Y) and the Gaussian variable (Z) are linked by the following anamorphosis function phi: $Z = phi\ (Y)$.

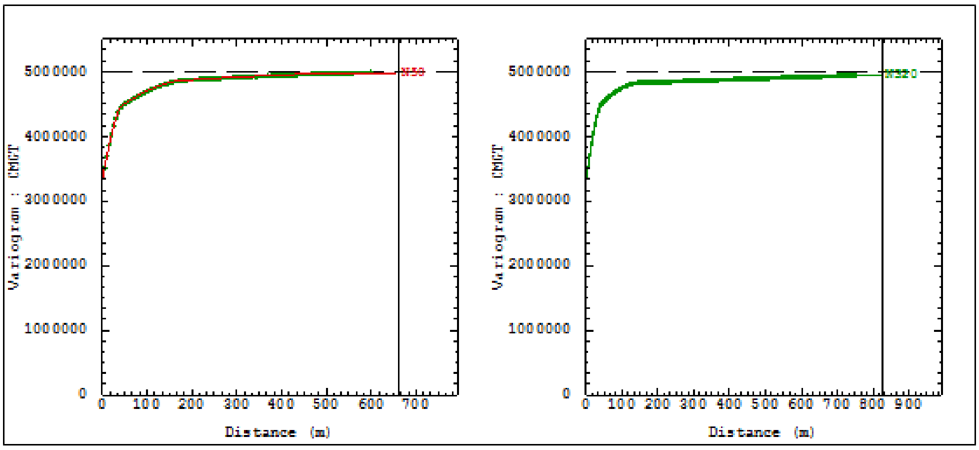

Figure 11: Back-transformed Cm.g/t experimental and model variogram for short (N50, left) and long (N140, right) directions

The normal scores and back-transformed variograms shown in Figure 10 and Figure 11 are robust, well-structured and easily modelled. Ranges for the first structures are about 40-50 m and the second structure has a range of about 200 m in both variograms indicating long distance grade continuity which is somewhat unusual in the type of deposit. However, the second range was not used during the estimation process as was defined in the neighbourhood kriging analysis (explained in the next section). The sills of the variogram model in different directions are only marginally different; a feature commonly found in channelised environments. The nugget effect is high which indicates that at short distances, the variable exhibits high spatial variation, typical of what is expected in a coarse gold deposit. The variogram model also reveals that the data are second order stationary, i.e. they come for the same domain and there is no trend in the dataset.

Quantitative Kriging Neighbourhood Analysis

The search neighbourhood refers to the number and spatial configuration of points to be used in estimation at an unmeasured location. The shape of the search ellipse is dictated by the anisotropy, having its major axis parallel to the direction of maximum spatial continuity (Isaaks and Srivastava, 1989). Quantitative Kriging Neighbourhood Analysis (QKNA), a method for determining the search neighbourhood (Vann *et al.,* 2003), was used to calculate the maximum number of samples used as well as the number of discretization points used for each block size (Figure 12). Beyond 55 samples there is no change in SLOR, KE or estimation variance (the line graphs flatten).

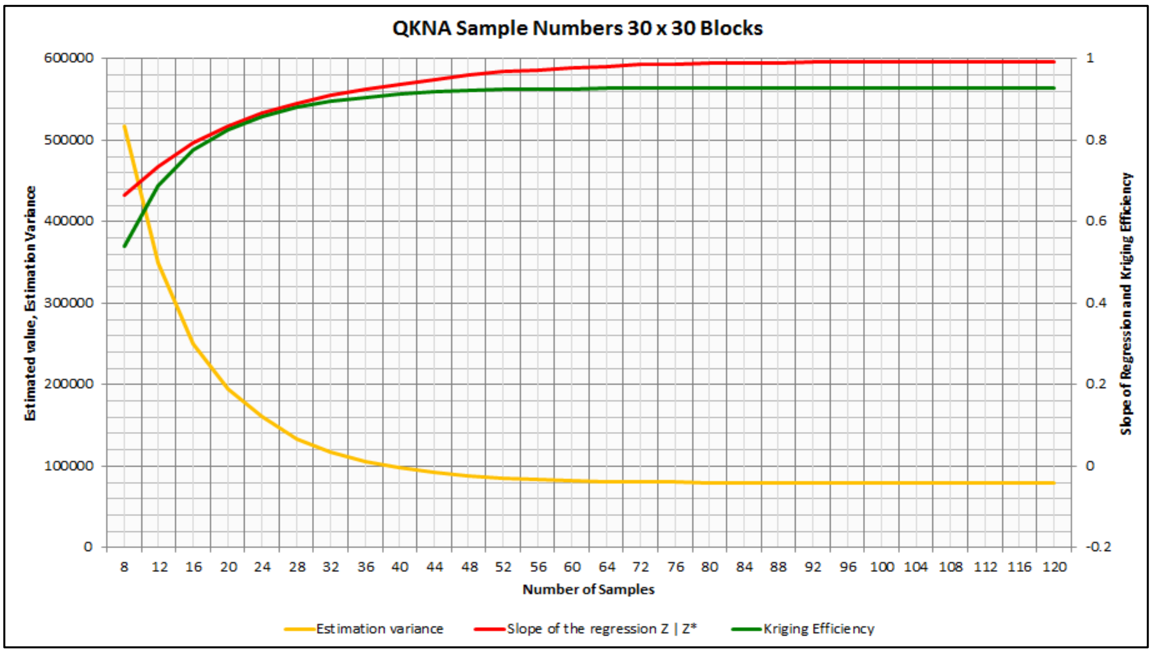

Figure 12: QKNA sample number optimisation graph

Curves for the slope of regression, (SLOR), kriging efficiencies (KE) and estimation variance shown Figure **12**, were determined on 30×30 m blocks using a quadrant search. Ideally, a value of one for the SLOR implies conditional unbiasedness, meaning that the true grade for a set of blocks should be approximately equal to the grade predicted by the kriged estimation (Vann *et al*., 2003). For the purpose of this exercise, 8 and 36 were chosen as the minimum and maximum number of samples for estimating a 30×30 m block. Increasing the number of samples above this will not significantly increase the SLOR or KE but could result in over-smoothing of the block estimates.

Block Discretization Number Optimisation

A block is regularly discretized according to the number of discretization points along X, Y and Z. The points along X, Y and Z axes are randomly and independently moved by off-setting the origin of the discretization grid and calculating each discretization several times. A graph of the range of $\bar{C}(V,V)$ (covariance) versus number of discretizing points is shown in Figure 13.

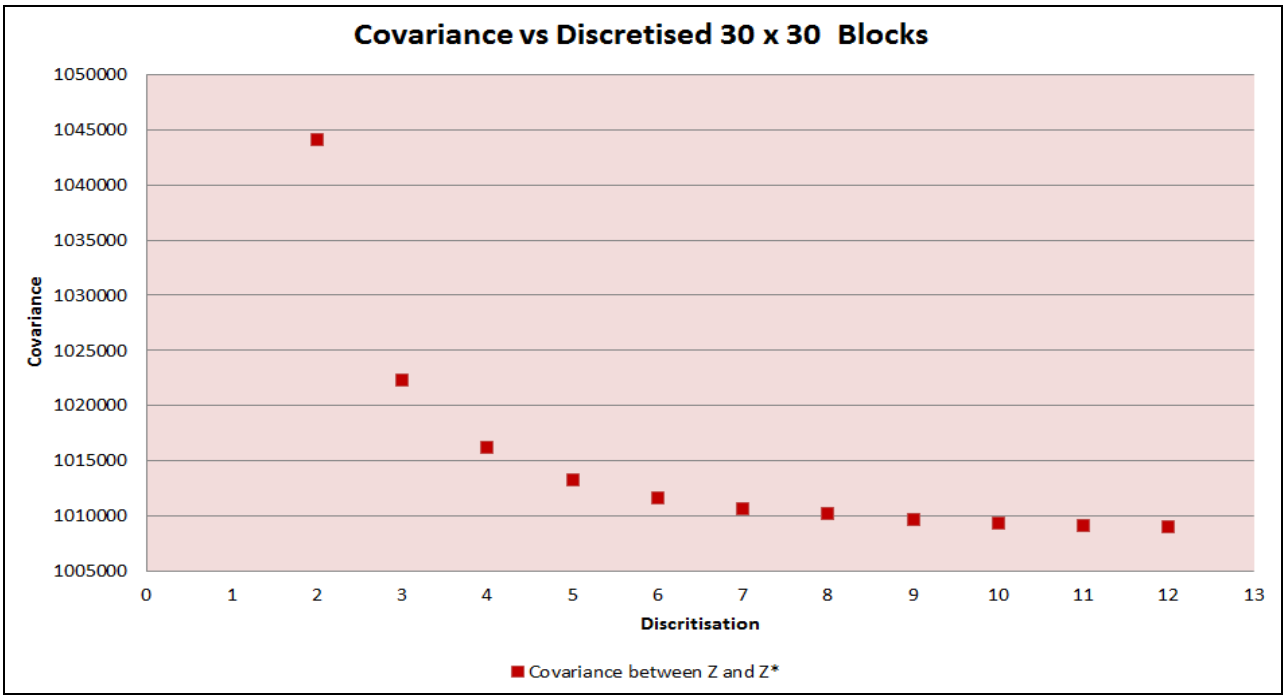

Figure 13: Graph showing sensitivity of covariance to block discretization.

According to Vann *et al*., (2003) stable results indicate that the discretization is adequate. The curve in Figure **13** show that the average covariance does not decrease significantly beyond about 7 or 8 discretizing points.

Dynamic Anisotropy Application on 30×30 m Block

Using Datamine Studio 3 software, strings (lines) were digitized and used to define the orientation of the mineralisation trend shown on Figure 14, which shows the sample location plan colour coded in Au cm.g/t. These lines followed the pay channel trend and are shown in Figure 15. The string file was input to the ANISOANG process command in Studio 3 to create a “points” file. Figure 15 shows the points as dotted lines and each “line” is a locally interpreted “direction”.

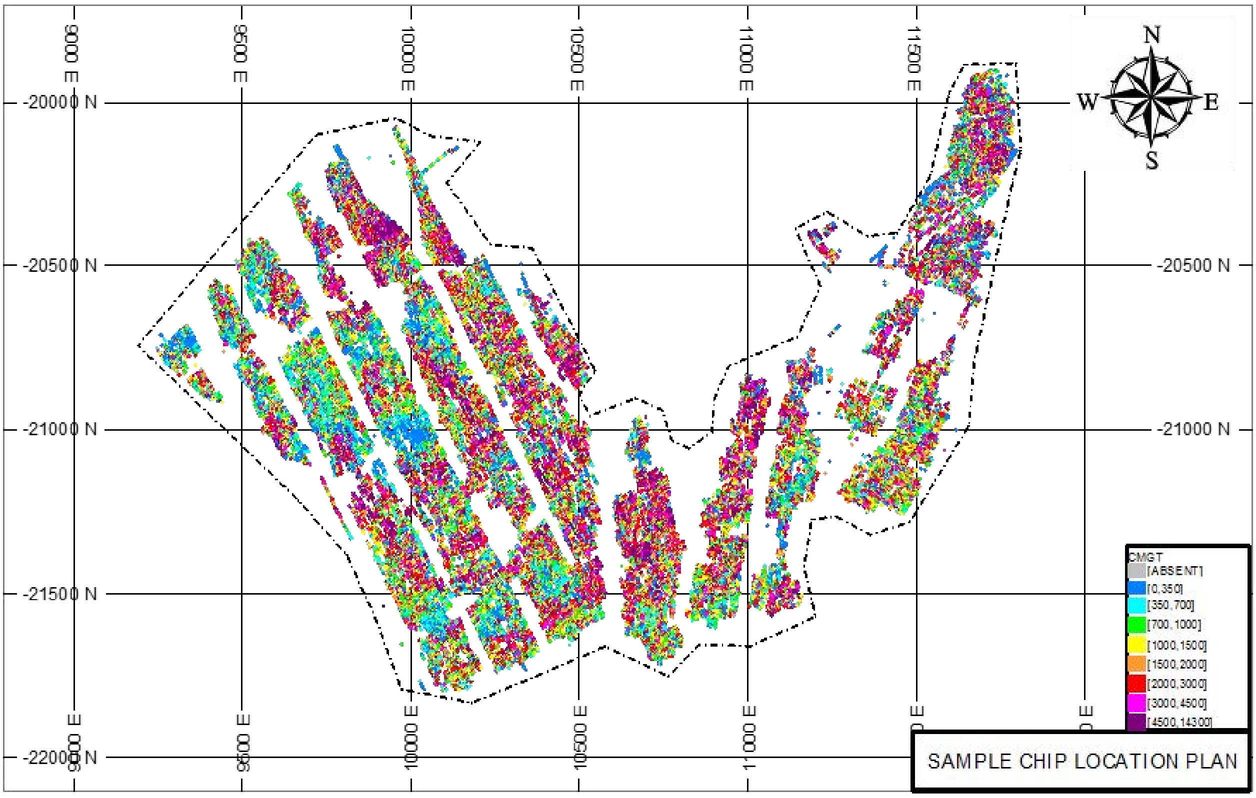

Figure 14: sample location plan colour coded in Au cm.g/t.

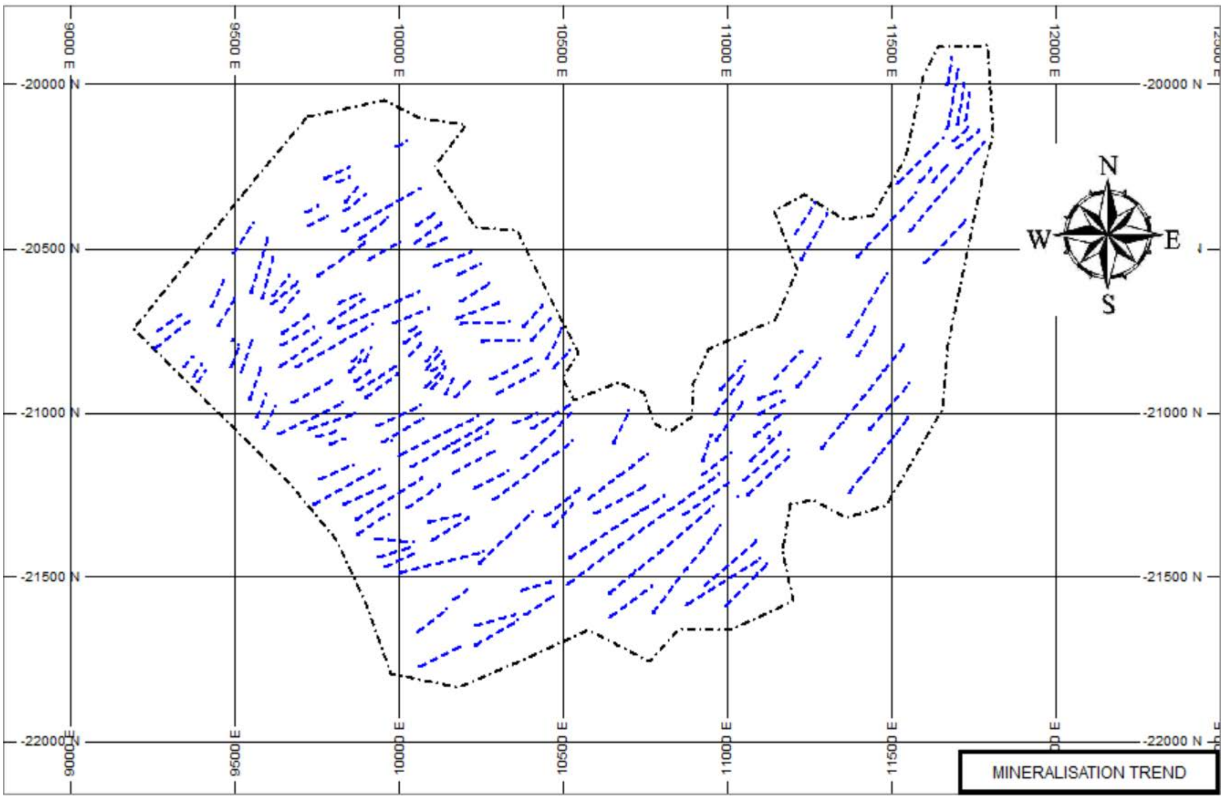

Figure 15: Orientation of mineralization (showing pay channel trend).

Using the "ESTIMA" process in Studio 3, the Inverse Power of Distance method of interpolation was used to interpolate the mineralisation trend direction data into a 30×30 m block model. The DA option in Studio 3 allows the anisotropy rotation angles for defining the search volume to be defined individually for each 30 x 30 m cell in the model. The search volume ellipse was oriented precisely and followed the trend of the mineralization. Since the estimation process was done in 2D, the dip was set at 0, and only the mineralisation trend direction was defined. The cmg/t value was then interpolated into the block model this time using the DA angles for the blocks instead of a set search ellipse. During the estimation process, the space around the centre of a block being estimated was divided into four quadrants by the axial planes of the data search ellipsoid. This parameter ensured that the samples informing an estimate were relatively evenly spread around the block.

Optimal DA Resolution Determination

In order to determine the optimal DA resolution for grade estimation improvement, the estimation process described above was executed on block model range of different resolutions which include; 30×30 m, 60×60 m, 120×120 m and 240×240 m grids for both estimation techniques, DA and OK. The resulting estimates were then compared against each other.

Results

Scatter Plots

The block estimates were validated against the input composite sample "source" data. The sample data was regularised where the average value of all samples within a grid cell of 30×30 m in X and Y directions was extracted and the mean statistic compared with corresponding block model estimate with similar block IJK values. The IJK field in a block model is an index value giving the "location" of the parent cell for each cell or subcell. All subcells in a parent cell have the same IJK value. Scatter plots showing the relationship between the block model estimates, and sample data are shown in Figure 16.

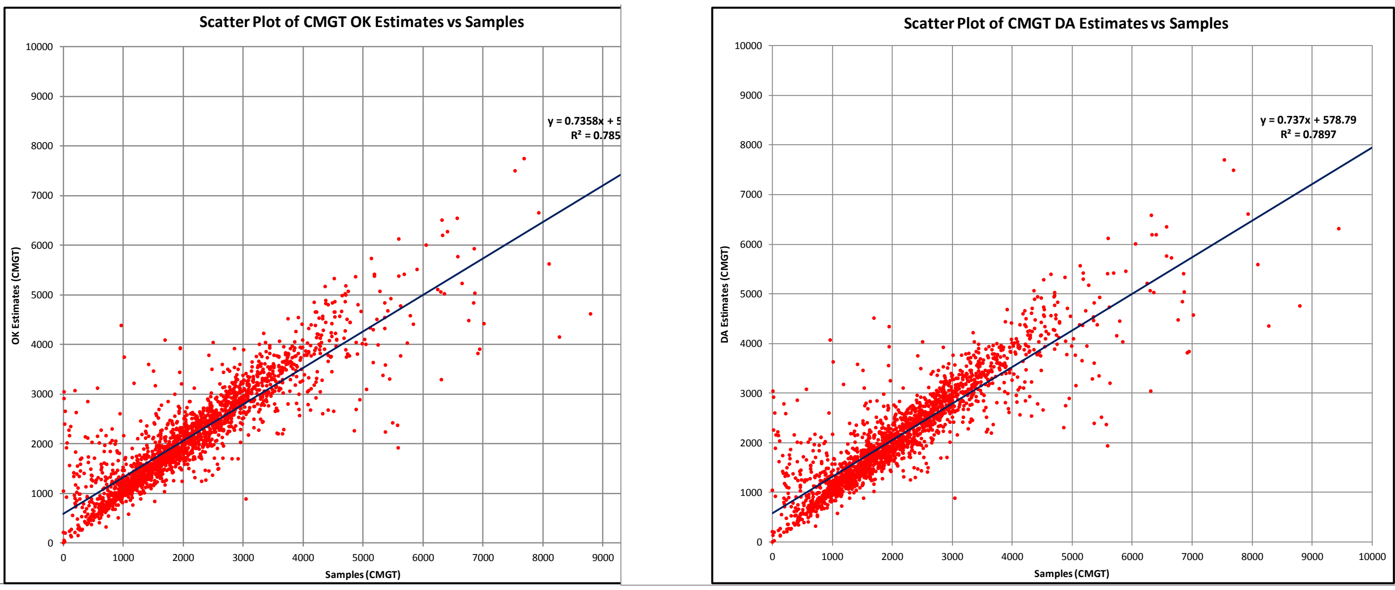

Figure 16: OK estimates (left) and DA estimates (right) vs. sample data scatter plots.

The strength and direction of the linear relationship between OK estimates and sample values and DA estimates and sample values was determined and the equation for the best-fit lines calculated using the "linear regression" function in EXCEL. Regression lines for the graphs in Figure 16 were calculated as follows:

$y = 0.736x + 580.85$ …………for sample data vs OK estimates
$y = 0.737x + 578.79$ …………for sample data vs DA estimates

Where x is the sample data and y is the estimate. When x=0, y=580.85 for OK estimates and 578.79 for DA estimates. However, the intercept is not considered in evaluating the performance of the estimator. The slope for both methods of interpolation is positive and is 0.736 and 0.737 for OK and DA estimates, respectively. This indicates that the estimates calculated from both methods of interpolation processes are similar, implying that when the sample grades increase by a factor of 1, the value of the corresponding estimates increases by a factor of 0.74 and a larger constant has to be added ~ 580 cmg/t for both OK and DA estimates. The correlation coefficient, R measures the precision, strength and direction of a linear relationship between two quantitative variables on a scatterplot. It is observed from Figure 16 that there is a positive and strong linear relationship between the sample composites and estimates produced by both interpolation methods; and that the relationship between samples and DA estimates is a little stronger with a correlation coefficient value of 0.889 in comparison to 0.886 for OK estimates. However, the percentage difference between the two estimation techniques is quite small, 0.003. Thus indicating that the application of DA estimation technique in 2D over the 30×30 m block model slightly improves the quality of estimates however, the improvement is not that not significant.

Discussion

To determine the optimal resolution that DA would provide an improvement in grade estimates, the results of estimation carried out in 60×60 m, 120×120 m and 240×240 m block sizes using DA and OK methods were compared. The maximum and minimum number of samples used for each of the different block sizes was determined using the QKNA approach suggested by Vann *et al*., (2003), and the resultant estimates validated. The sample composites in each of the different block sizes were regularised and the average value compared with the corresponding block estimate. The slopes and correlation coefficients between the averaged block grade and the kriged estimates determined using DA and OK methods for the various block sizes, are listed in Table 2.

Table 2: Line of regression equations and correlation coefficients on blocks.

Block	Est Method	R 2	R	Regression equation
30 × 30	DA	0.79	0.889	y = 0.7370x + 578.79
30 × 30	Normal OK	0.785	0.886	y = 0.7358x + 580.85
60 × 60	DA	0.797	0.893	y = 0.8760x + 284.68
60 × 60	Normal OK	0.797	0.893	y = 0.8793x + 280.83
120 × 120	DA	0.615	0.784	y = 0.8734x + 209.99
120 × 120	Normal OK	0.612	0.782	y = 0.8709x + 214.42
240 × 240	DA	0.26	0.51	y = 0.9777x + 65.710
240 × 240	Normal OK	0.295	0.543	y = 1.0138x - 47.016

The slope and correlation coefficients for different block sizes using DA and OK estimation methods are plotted in Figure 17 and Figure 18 respectively.

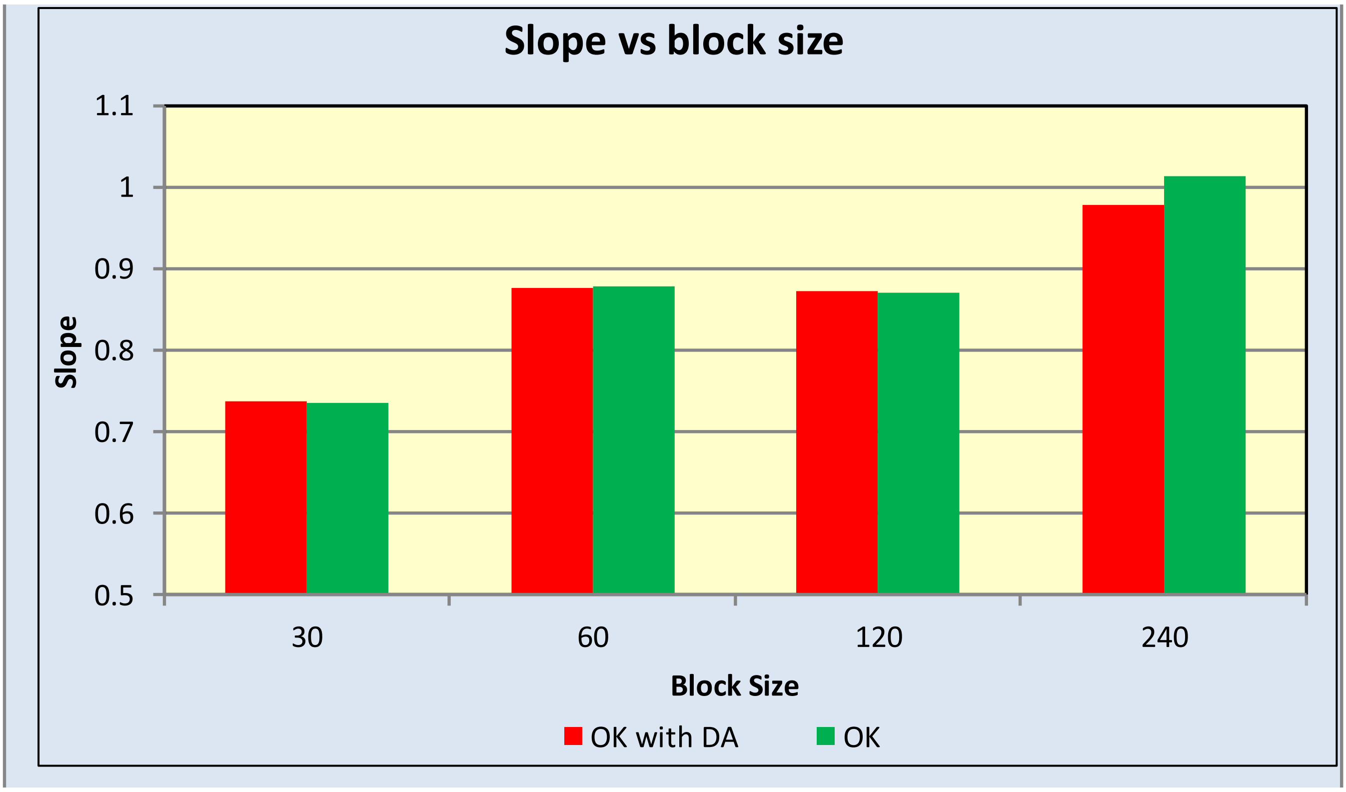

Figure 17: Slope against block size.

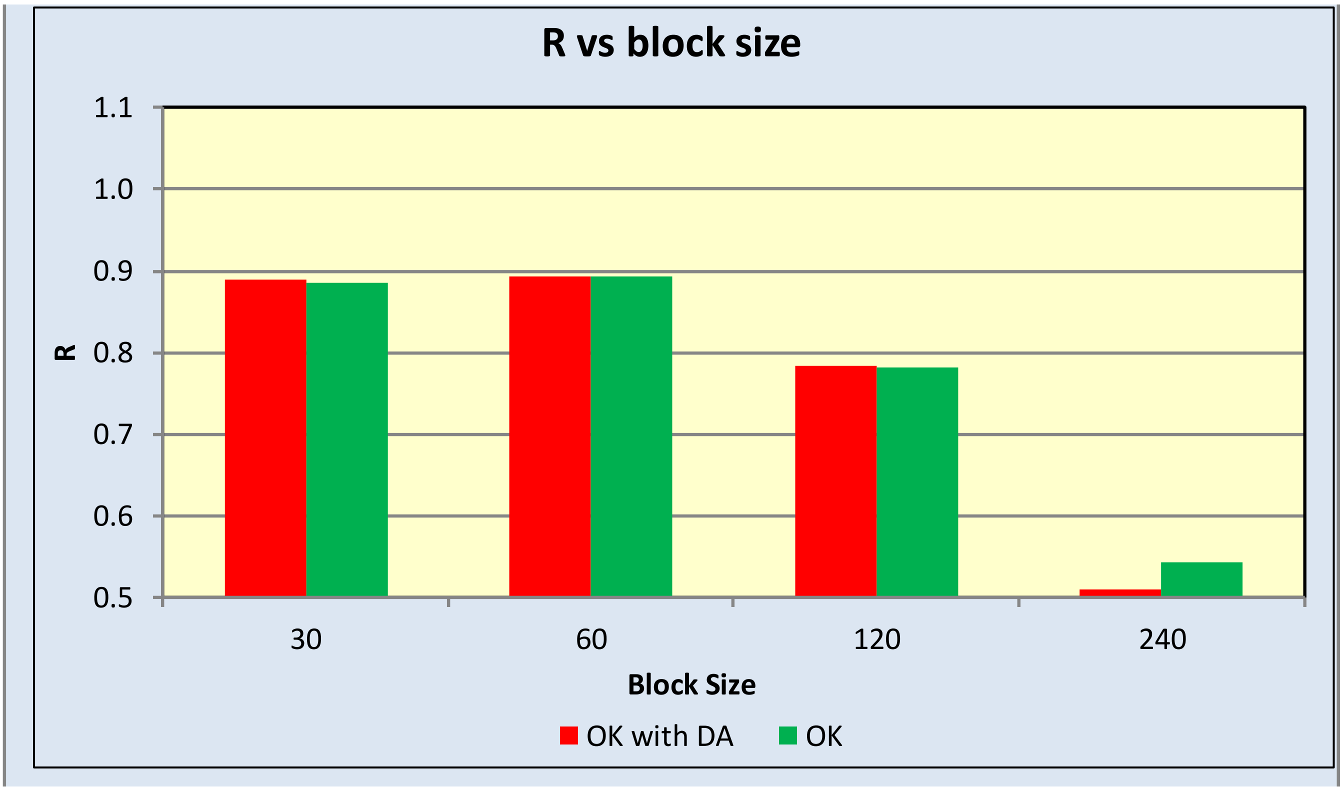

Figure 18: Correlation coefficient against block size.

The following is deduced from Table 2, Figure 17 and Figure 18:

i) Application of DA in 2D over the 30×30 m block model (Case Study) slightly improves the quality of estimates however, the improvement is not that not significant;

ii) Slope increases significantly with increasing block size which implies that increasing the block size increases the quality of estimates for both interpolation methods, however R decreases with increasing block size implying that the relationship between the samples and estimates weakens with increasing block size;

iii) Slope for the 240×240 m grid block is greater than one implying that there is a conditional bias of the estimates;

iv) There is a small to insignificant difference between the slopes for both methods executed on same block size. The study was executed in a 2D environment where the dip component of DA has been ignored. This is a limitation for this exercise and further explains why the results between OK and DA estimation techniques are very similar. Also, the dense spacing of the data in the blocks estimated precludes getting very different answers regardless of the technique used; and

v) The traditional estimation technique, which is OK, becomes a better method of estimation with increasing block size in comparison to the DA method of estimation. This can also be explained pictorially in Figure 19, which shows that as the block sizes increase, the anisotropy tends toward the regional anisotropy and the localized directions become misleading. In other words, as the block sizes approach the level of resolution of DA, the benefits thereof are reduced and as observed may even become a confounding factor in estimation leading to less accurate and smoothing of estimates.

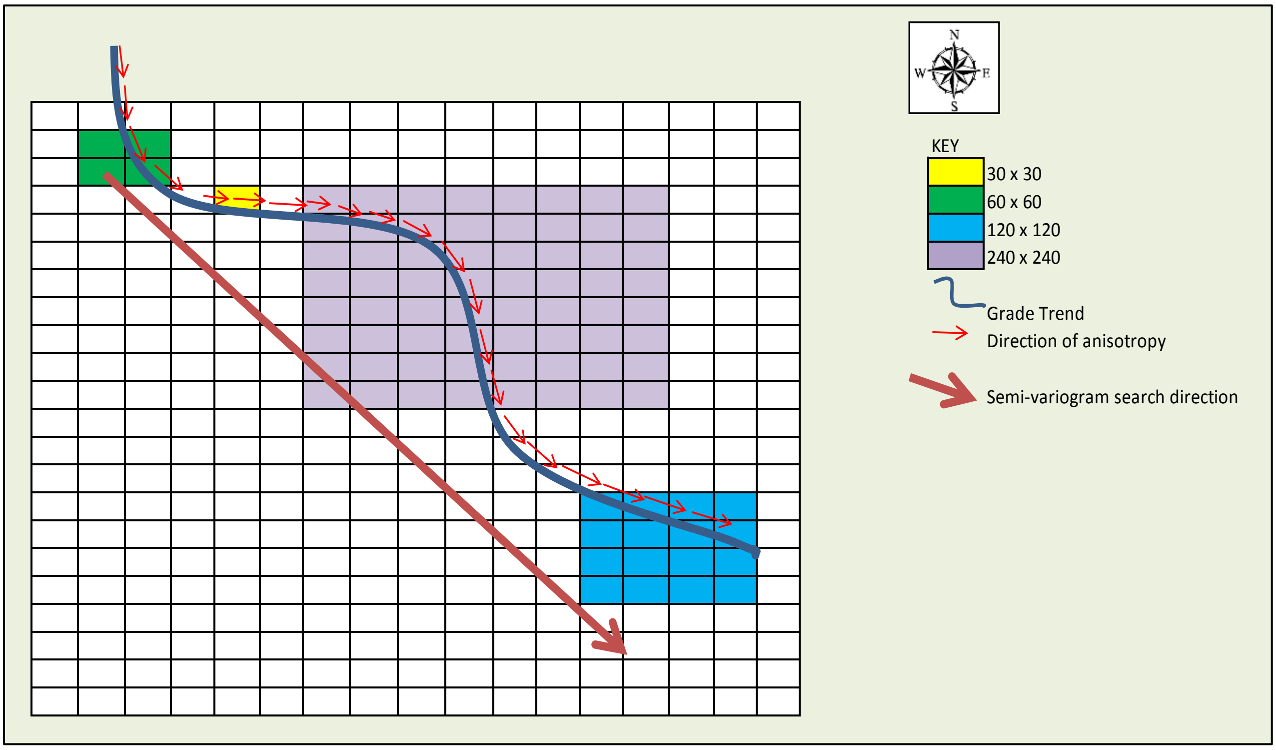

Figure 19: Diagrammatic representation showing relationship between block sizes, "resolution of Dynamic Anisotropy" and localised directions.

Conclusions

Accurately accounting for the continuity in grade in meandering depositional environments that are typical of the Witwatersrand type gold deposits using the DA method is considered to provide consistently improved estimates than if the direction of anisotropy is not taken into account. Rather than performing domain-based analysis with different search ellipses, DA can be applied during the estimation process whereby the search volume is oriented to follow the trend of mineralization precisely. A dynamic search ellipse that automatically adjusts to the direction of maximum continuity allows for a truer estimation of the grade in geological settings characterised by undulating, meandering or folded host rock structures. The DA method identifies the data to be used in the estimation on a block by block basis, allowing the direction of the search ellipsoid to change along strike according to local variations in the dip and dip direction for each block. Thus, DA improves sample selection and kriging efficiency during the estimation of grade.

This paper confirms that the DA approach provides a slight improved estimation of grade in an area of the Driefontein Gold mine lying on a slightly undulating, gently folded limb of a syncline. DA and OK interpolation techniques were applied to the data set extracted from the case study area and the resultant estimates compared. It was observed that the application of DA estimation technique in 2D over the 30×30 m block model slightly improves the quality of estimates however, the improvement is not that not significant.

This is attributed to the fact that for smaller block sizes, samples being closer together tend to share weights resulting in estimates that are very similar, whereas this is not the case with larger block sizes.

The optimal block size for improving grade estimates was determined using the estimates for grade following a range of block sizes; 30×30 m, 60×60 m, 120×120 m and 240×240 m. Analysis of the slope (gradient) and the kriging efficiency indicate that the gradient increased significantly whilst the correlation coefficient, R decreased with increasing block size. This implies that increasing the block size improves the quality of estimates for both DA and OK, but reduces R, suggesting that the correlation between the sample data and block estimates weakens as the block size increases. This may be due to the fact that there are more samples within a block than are used to calculate the estimate. Where a large block contains more samples than are used for estimation, using the average is probably a better estimate of grade than can be produced by kriging. The results also revealed that with increasing block size, OK is a better estimator of grade compared to DA because at these scales, local anisotropy tends toward the regional anisotropy and localized directions become misleading.

A limitation for this study which may explain the similarity between OK and DA estimation techniques is that the 2D study only took account of direction of thee mineralisation trend and not the dip; as the dip component of DA was not used. A similar exercise based on three dimensional and widely spaced, regularised point data and Simple Kriging is planned as a future study at Driefontein Gold Mine. In addition, further investigations in an exploration area with low sample data density are intended to test whether the DA method of interpolation will improve the quality of estimates.

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