RELIABILITY ANALYSES FOR GROUNDWATER FLOW IN OPEN PIT MINES – QUANTITY OF FLOW

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ABSTRACT

Seepage considerations are a key element in analyzing the geotechnical stability performance of soil and rock slopes in open pit mines. Two effects need to be considered: the quantity of flow (operational) and the reduction in the soil/rock mass shear strength resulting from water pore pressures within the rock and/or soil masses. Groundwater flow through soil/rock masses is mainly controlled by rock mass geometry and discontinuities, material properties (i.e. hydraulic conductivity, permeability) and geological characteristics of the rock mass. All of these factors have large uncertainties because soil/rock masses are usually heterogeneous and anisotropic.

The objective of this paper is to develop practical equations and methodologies to estimate the quantity of flow within the slopes of open pit mines. Plane closed form solutions and finite element numerical models are utilized to estimate the quantity of seepage for different slope geometries and boundary conditions. Probability analyses can be developed applying the FOSM method to understand the uncertainty on the quantity of flow due to assumptions in the probability distributions for hydraulic conductivity and total head for a given slope geometry. These results were compared with Latin hyper-cube simulations using spreadsheet and @Risk software for several statistical distributions of hydraulic conductivity and total head.

INTRODUCTION

Regional tectonic and in situ stresses and other geological variables control the occurrence of joints, fractures, and shear zones which are the main mechanism for fluid flow in a fractured rock mass. If joints and fractures are open and connected as they often are in open pit mines, they control the quantity and direction of flow. In addition, hydraulic conductivity of fractures or joints can vary by orders of magnitude depending on their aperture, porosity, infilling material and roughness. Hence, assumptions of homogeneity and isotropy are seldom suitable for the description and quantification of fluid flow in fractured rock even though it is the preferred engineering tool.

Since intact rock "permeability" is low, low discharges are expected for intact rock (pore pressures can still be high). On the other hand, if the rock is discontinuous as a result of the presence of joints, faults, fissures and other discontinuities, the permeability can be considerably higher because these discontinuities act as channels for the flow of water. Hence, the permeability of fractures and fissures in rock generally can be orders of magnitude greater than the permeability of intact rock, and the seepage characteristics are mainly controlled by the nature and spacing of the discontinuities within the rock mass. As an example, in some metamorphic and intrusive igneous rocks (such as schist and basalt) the permeability of intact rock is usually negligible in comparison with that of the fissures (Sharp et al., 1972).

The influence of groundwater (seepage) on the stability of rock masses has been studied as lessons learned from a number of large slope and dam failures (among others, Harr, 1962; Cedergren, 1967 & González de Vallejo, et al., 2002). Water pressures acting within discontinuities in the rock masses reduce effective stresses with a consequent reduction in shear strength. Given the current state of knowledge and practice in geotechnical engineering, it is difficult to precisely model groundwater flow in a jointed rock mass. Consequently, the current approach is to perform parametric studies of extreme, but realistic cases, to bracket the range of possible seepage and to assess the sensitivity of the slope to variations in groundwater conditions (Hoek & Bray, 1981). There are two main approaches to determining the quantity of flow within a rock mass:

a) Numerical techniques (i.e. finite elements, finite differences) to model the flow pattern based on slope geometry, material properties (e.g. hydraulic conductivity), recharges, and visual field observations.

b) Field measurements (i.e. piezometers and observation wells) to determine pore pressures using analytical solutions and flow nets[1] with limited use of numerical techniques.

Both methods have practical difficulties but, because of the important influence of water in open pit mines, it is essential that the best possible estimates of these quantities be performed.

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[1] A flow net is a graphical solution of Laplace's equation for homogeneous and isotropic soil conditions, and consists of a plot of lines of equal total hydraulic head, called equipotentials, and streamlines or flow lines. Flow nets can be constructed graphically, by trial and error, to any desired accuracy and to satisfy even complex geometries and boundary conditions. This technique was developed by Forchheimer (1930), and a classical summary was presented by Casagrande (1937). Nevertheless, the flexibility and power of numerical methods, especially finite element methods, have made them the techniques of choice for analysis of groundwater flow, and to a large extent they now supplant hand-drawn flow nets (Hormazábal et al. 2005).

BACKGROUND

CONFINED FLOW[2]

In 1856, Henry Darcy published, in a report on the construction of the Dijon, France municipal water system, a relationship for the flow rate of water in sand filters. In the report, Darcy describes a laboratory experiment to analyze the *confined flow*. The result of this experiment is the well known Darcy's law. The experiments carried out by Darcy (see **Figure 1**) showed that **v** (discharge velocity) is directly proportional to the difference in fluid levels **h_1** - **h_2** ($v \propto \Delta h$) when **L** (soil sample length) is constant and inversely proportional to **ΔL** when **h_1** - **h_2** is constant ($v \propto 1/\Delta L$). Darcy's law can be written as:

$$v = k\frac{\Delta h}{\Delta l} \quad \text{or (in the limit, as } \Delta L \to 0\text{)} \quad v = k\frac{dh}{dl} \tag{1}$$

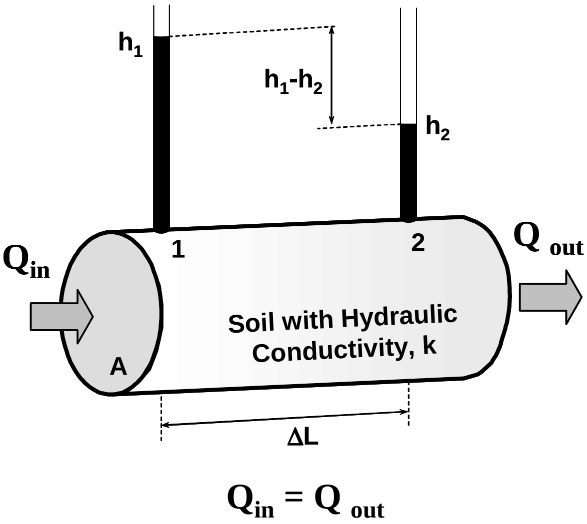

Figure 1.- Experimental apparatus for the illustration of Darcy's law.

In Eq. (1), **h** is called the hydraulic head and **dh/dl** is the hydraulic gradient. The parameter **k** is the constant of proportionality and is known as the hydraulic conductivity[3]. It has high values for sands and gravels and low values for clays and most rocks (Casagrande, 1937, Freeze & Cherry, 1979 among others). An alternative form of Darcy's law can be obtained by substituting v = Q/A in Eq. (1) to yield:

$$Q = k\frac{dh}{dl}A \quad \text{or} \quad Q = kiA \tag{2}$$

where **i** is the hydraulic gradient.

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[2] Confined flow is considered as the groundwater flows in a confined aquifer bounded above and below by confining units of distinctly lower permeability than that of the aquifer itself (ASCE, 1985).

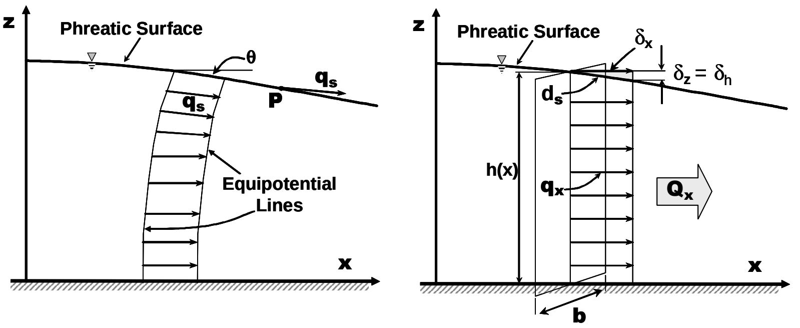
[3] In geotechnical engineering this parameter is considered as permeability.

Darcy's law is an empirical law, and assumes that the flow is laminar[4] and the soil is saturated. Confined groundwater flow is governed by Darcy's law (Eq. (2)). Flow through a porous medium under steady state conditions in two dimensions is described by the following equation – Laplace's equation:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right)=0 \tag{3}$$

UNCONFINED FLOW[5]

In 1863, Jules Dupuit solved the problem of steady flow in an *unconfined aquifer* (see **Figure 2**).



(a) Real Case **(b) Dupuit's assumptions**

Figure 2.- Two dimensional problem for unconfined flow.

The assumptions are that (1) the hydraulic gradient is equal to the slope of the water table and (2) for a small water table gradient, the streamlines or flowlines are horizontal and the equipotential[6] lines are vertical. In steady two-dimensional unconfined flow, without addition in the vertical xz plane, the phreatic surface is a streamline. At every point along of this phreatic surface, the specific discharge qs, (see **Figure 2a**) is given by Darcy's law:

$$q_s = -k\frac{dh}{ds} = -k\frac{dz}{ds} = -k\sin\theta \tag{4}$$

As θ is very small, Dupuit suggested that sinθ be replaced by the slope tanθ = dh/dx. The assumption of a small θ is equivalent to assuming that equipotential surfaces are vertical (i.e., h=h(x) is independent of z) and that flow is essentially horizontal. Thus, the Dupuit assumptions lead to specific discharge expressed by:

$$q_s = -k\frac{dh}{dx}; \qquad h=h(x) \tag{5}$$

And for the total quantity of flow through any vertical surface of width b (see **Figure 2b**):

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[4] Since Darcy's law states that the velocity of flow is proportional to the first power of the hydraulic gradient, Darcy's equation is valid only for laminar flow (Fetter, 2001).

[5] Unconfined flow is considered as an aquifer in which the water table forms the upper boundary and occurs near the ground surface.

[6] Equipotential lines are lines of equal energy level or hydraulic head.

$$Q_x = -kbh(x)\frac{dh}{dx}\,; \tag{6}$$

The quantity of flow and the line of seepage can be determined using Darcy's law and Laplace's equation for homogeneous and isotropic conditions:

$$Q = kiA \qquad \text{or} \qquad Q = k\frac{dh}{dx}h \quad \Rightarrow \quad \frac{Q}{k}dx = hdh \tag{7}$$

with boundary conditions at x = 0 when h = h_1 and x = d_0 when h = h_2; we have:

$$\frac{Q}{k}\int_0^{d_0} dx = \int_{h_1}^{h_2} hdh \qquad \Rightarrow \qquad \frac{Q}{k}d_0 = \frac{h_2^{\,2} - h_1^{\,2}}{2} \tag{8}$$

The quantity of flow can be written as:

$$Q = k\left[\frac{h_2^{\,2} - h_1^{\,2}}{2d_0}\right] \tag{9}$$

METHODOLOGY

ESTIMATION OF QUANTITY OF FLOW IN SLOPES

Figure 3 shows the geometry of a simplified cross section of an open pit mine slope. The geometry is divided into two portions through the use of an "impermeable virtual boundary". These two components of flow will be defined as the Dupuit and the Darcy. Then, the flow is determined using the principle of superposition (Hormazábal, 2005).

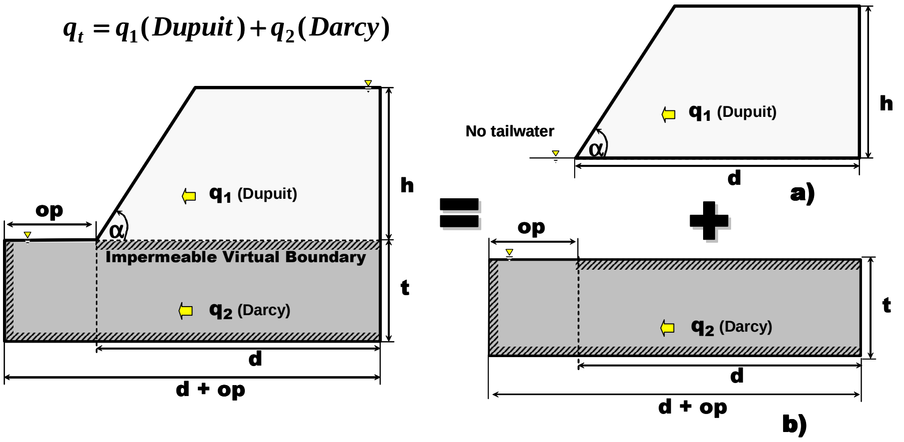

Figure 3.- Methodology applied to estimate the total quantity of flow; $\mathbf{q_t}$.

Plane Solution for Isotropic and Homogeneous Case

Dupuit's component (q_1)

Applying the Dupuit solution to a slope case and using Eq. (9), h_2 = h, h_1 = 0 and d_0 = d (see **Figure 3a**), the quantity of flow can be expressed in the following form:

$$\frac{q_1}{k}d = \frac{h^2}{2} \qquad \Rightarrow \qquad q_1 = \frac{kh^2}{2d} \tag{10}$$

Darcy's component (q₂)

For this case, the equation for confined flow was derived considering a series of analyses of finite element numerical models. These finite element numerical models were run with values of op/t varying between 0.05 and 1.5, for several combinations of t/d (**Figure 3b**). Regression analyses were then applied to the results, and we found that for the confined flow of Darcy's component, the following expression can be used:

$$\frac{kh}{q_2} = \alpha + \frac{d}{t} \qquad \text{or} \qquad q_2 = \frac{kh}{\alpha + \frac{d}{t}} \tag{11}$$

with

$\alpha = 0.44$ for op/t > 0.5 $\qquad$ $\alpha = 0.2 - \log(op/t)$ for op/t ≤ 0.5

Plane Solution for Anisotropic Case

For the anisotropic case, the above results are easily extended to include the effects of anisotropy[7]. The procedure is the conventional technique of the transformed section (see **Figure 4**). The steps are:

1) Use the transformed x coordinate $x_T = x_N(k_y/k_x)^{1/2}$. x_N represents the "natural scale"[8].
2) Determine the equivalent hydraulic conductivity $k_{EQ} = (k_x k_y)^{1/2}$.
3) Use Eq.(10) and Eq.(11), with the transformed geometry and k_{EQ} to estimate the quantity of flow for the Dupuit and Darcy components.

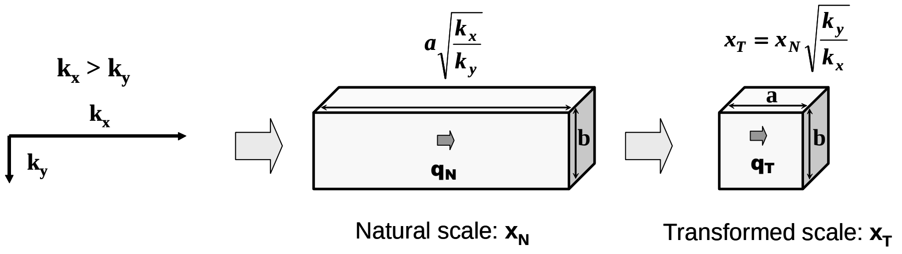

Figure 4.- Conventional technique of the transformed section.

Using Darcy's law and the above procedure, the quantity of flow per unit length at natural and transformed scale can be written as:

$$q_N = k_x \frac{\Delta h}{a\sqrt{\frac{k_x}{k_y}}} b \times 1; \qquad q_T = k_{EQ}\frac{\Delta h}{a} b \times 1 \tag{12}$$

because $q_T = q_N$,

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[7] If the principal directions of anisotropy are not horizontal and vertical or if the rock masses/soils are not homogeneous, such simple methods are not applicable.

[8] Natural scale is considered as the real dimension or distance in the horizontal direction.

$$k_{EQ}\frac{\Delta h}{a}b = k_x\frac{\Delta h}{a\sqrt{\frac{k_y}{k_x}}}b \Rightarrow \qquad k_{EQ}=\sqrt{k_x k_y} \tag{13}$$

Dupuit's component (q_1)

Replacing Eq.(13) and the new x coordinate x_T in Eq. (10):

$$q_1 = \frac{\sqrt{k_x k_y}\,h^2}{2d\sqrt{\frac{k_y}{k_x}}} = \frac{k_x h^2}{2d} \tag{14}$$

Darcy's component (q_2)

Using the same procedure, the quantity of flow for the Darcy component, Eq. (11), can be written as:

$$\frac{h\sqrt{k_x k_y}}{q_2} = \alpha + \frac{d}{t}\sqrt{\frac{k_y}{k_x}} \qquad \Rightarrow \qquad q_2 = \frac{ht\sqrt{k_x k_y}}{d\sqrt{\frac{k_y}{k_x}} + \alpha t} \tag{15}$$

where

$\alpha = 0.44$ for $op_T/t > 0.5$; $\alpha = 0.2 - \log\frac{op_T}{t}$ for $op_T/t \le 0.5$; and $op_T = op\sqrt{\frac{k_y}{k_x}}$

Plane Solution for Non-Homogeneous Case

Figure 5 shows the geometry used to derive an expression for the non-homogeneous case. It is assumed that a vertical material of thickness "w", and hydraulic conductivity "k_2" simulate a dyke or shear zone. The objective is to find the equivalent thickness "w_t" with hydraulic conductivity "k_1" ($k_2 \neq k_1$).

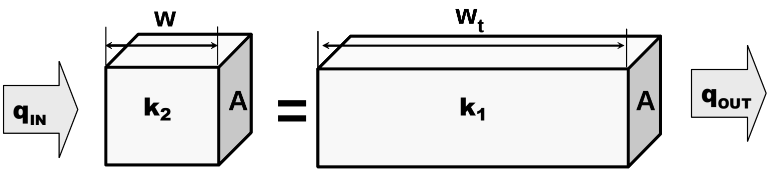

Figure 5.- Geometry used to derive an expression for the quantity of flow of non-homogeneous material.

By continuity q_{IN} is equal to q_{OUT}. Thus,

$$q_{IN} = k_2\frac{h}{w}A = k_1\frac{h}{w_t}A = q_{OUT} \tag{16}$$

and,

$$\frac{k_2}{w} = \frac{k_1}{w_t} \qquad \text{or} \qquad w_t = \frac{k_1}{k_2}w \tag{16}$$

Thus, knowing the value of w_t, we can redefine the new distance "d_t":

$$d_t = d - w + \frac{k_1}{k_2} w \text{ or } \qquad d_t = d + w\left[\frac{k_1}{k_2} - 1\right] \tag{17}$$

Dupuit's component (q$_1$)

Replacing Eq.(17) in Eq.(10):

$$q_1 = \frac{k_1 h^2}{2d_T} \qquad \text{and} \qquad d_T = d + w\left[\frac{k_1}{k_2} - 1\right] \tag{18}$$

Darcy's component (q$_2$)

The same procedure is utilized in determining the quantity of flow for the Darcy component, so Eq. (11) can be written as:

$$q_2 = \frac{k_1 ht}{d_T + \alpha t} \tag{19}$$

with

$\alpha = 0.44$ for op/t > 0.5; $\alpha = 0.2 - \log\frac{op}{t}$ for op/t ≤ 0.5; and $d_T = d + w\left[\frac{k_1}{k_2} - 1\right]$

Figure 6 compares the normalized quantity of flow using Eq.(10) and Eq.(11) with several analyses employing the finite element numerical models (Phase^2 6.0 and FeFlow 5.1). **Figure 7** shows the quantity of flow using Eq.(14) and Eq.(15); and numerical calculations of finite element models, an anisotropy ratio of $k_x/k_y = 4$ and $k_x/k_y = 9$ was considered for the anisotropic case. **Figure 8** shows the calculated quantity of flow using Eq.(18) and Eq.(19); and numerical solutions of finite element models, a hydraulic conductivity ratio of $k_1/k_2 = 100$ and $k_1/k_2 = 1000$, and thickness w of 2, 5, 10, 15 20 and 30 m was considered. All cases were considered for a slope angle $\beta = 45°$, a common value encountered in open pit mines[9].

ESTIMATION OF UNCERTAINTY OF QUANTITY OF FLOW

Uncertainly in geotechnical parameters involves not only index properties such as strength or hydraulic conductivity but also geometrical ones such as joint or fault inclinations and temporally varying ones such as total heads. Thus it is necessary to establish which parameters and models are important and to estimate the parameters as closely as possible. Reliability analyses have become a popular tool for geotechnical engineering. Most of the applications use the First-Order Second Moment (FOSM) method, which is easy to use and gives direct insight into parametric effects (Christian, 1994, Urzua et al, 2006).

Considering the large uncertainty associated with the hydraulic conductivity and total head, probability analyses can be developed applying the FOSM method to understand the uncertainty on the quantity of flow due to assumptions in the probability distributions for hydraulic conductivity and total head for a given slope geometry.

The First Order Second Moment (FOSM) method estimates the mean and the standard deviation of a function (quantity of flow) by integrating this function over the expected range of values of the random variables. As a result, variations and uncertainly in the random variables are explicitly accounted for (Riela, 2000).

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[9] Several finite element numerical models were run for values of β between 30° and 90° and compared to the Dupuit analytical solution and the results indicates differences less than 5% for slopes with $\beta \geq 45°$.

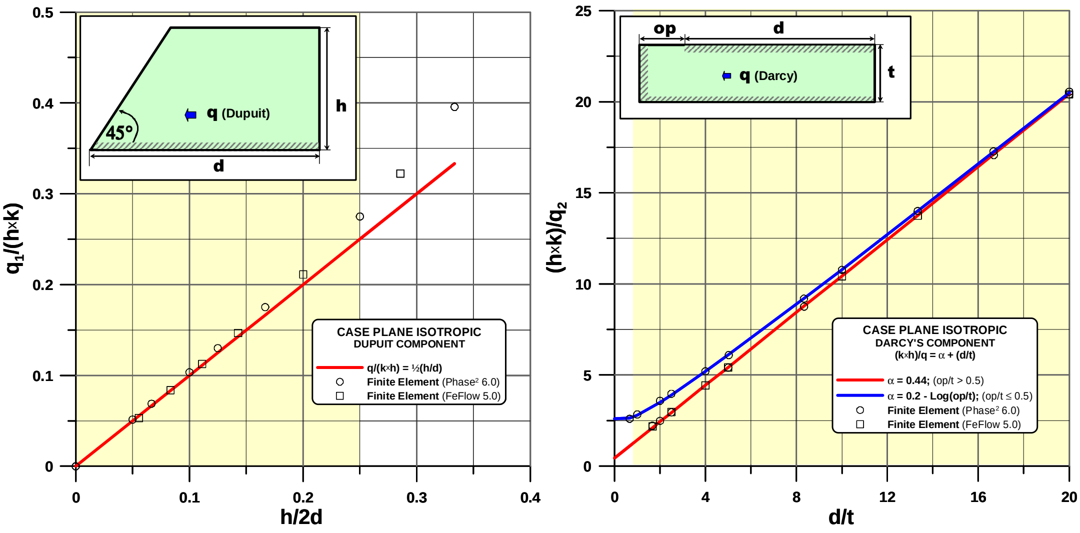

Figure 6: Quantity of flow normalized by total head and hydraulic conductivity for the Dupuit's and Darcy component, case plane, isotropic and homogeneous. Shaded zone indicates recommended application range.

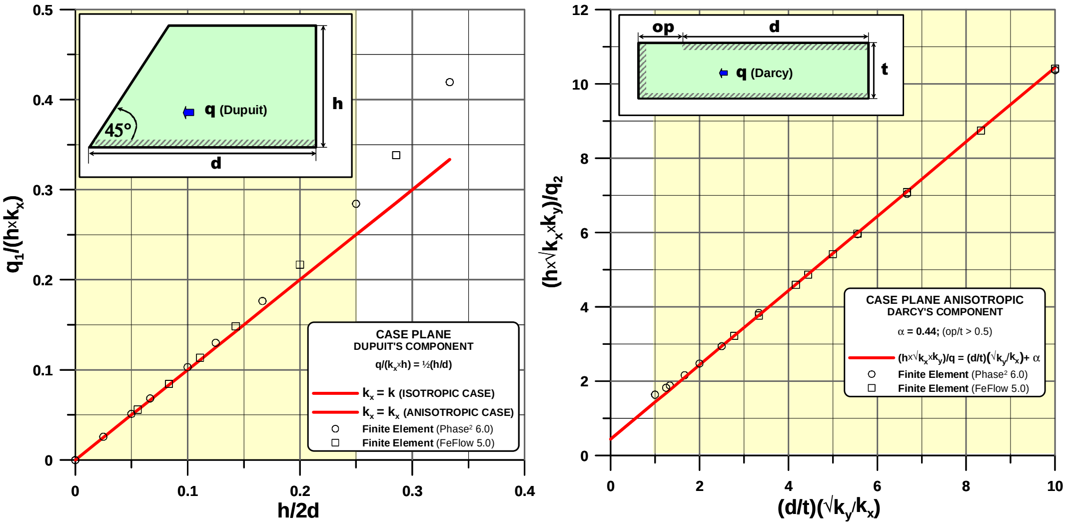

Figure 7: Quantity of flow normalized by total head and hydraulic conductivity for the Dupuit and Darcy components, case plane, anisotropic and homogeneous. An anisotropy ratio of $k_x/k_y = 4$ and $k_x/k_y = 9$ were considered. Shaded zone indicates recommended application range.

The FOSM method, in the forms of Eq. (20), leads to a particularly straightforward computational procedure (Christian, 1994). The computation requires that the variance of each variable and the partial derivatives be computed (Larsen & Marx (2001).

$$\sigma_E^2 \approx \sum_{i=1}^{n} \sigma_{x_i}^2 \left(\frac{\partial q}{\partial x_i} \right)^2 \tag{20}$$

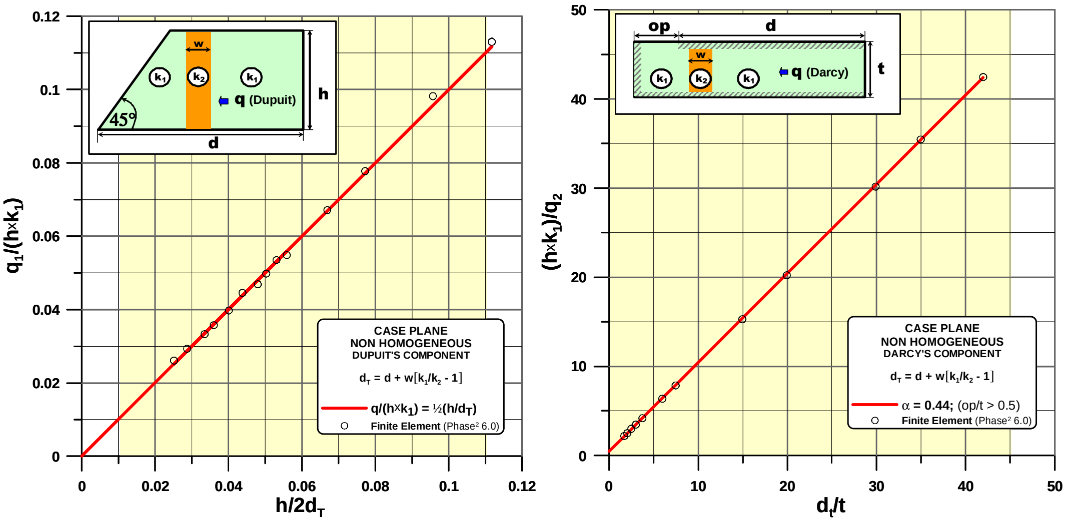

Figure 8: Quantity of flow normalized by total head and hydraulic conductivity, k_1, for the Dupuit and Darcy components, case plane, isotropic and non-homogeneous. Shaded zone indicates recommended application range. A hydraulic conductivity ratio of $k_1/k_2 = 100$ and $k_1/k_2 = 1000$, and thickness w of 2, 5, 10, 15 20 and 30 m were considered. Shaded zone indicates recommended application range.

Plane Isotropic Case

Expressing the quantity of flow in the following form:

$$q = \frac{h^2 k}{2d} + \frac{kht}{\alpha t + d}, \qquad \text{with} \qquad \alpha = 0.44 \rightarrow op/t > 0.5 \qquad (21)$$

Assuming uncorrelated variables as a first approximation, three variables must been considered[10]: k, h and t, with mean values μ_k, μ_h and μ_t, and standard deviations σ_k, σ_h and σ_t, respectively. Thus:

$$E[q] \cong \mu_k \left[\frac{\mu_h^2}{2d} + \frac{\mu_h \mu_t}{d + \alpha \mu_t} \right], \text{ with} \qquad \alpha = 0.44 \rightarrow op/t > 0.5 \qquad (22)$$

$$VAR[q] \cong (\sigma_k)^2 \left[\frac{h^2}{2d} + \frac{ht}{d + \alpha t} \right]^2 + (\sigma_h)^2 \left[\frac{kh}{d} + \frac{kt}{d + \alpha t} \right]^2 + (\sigma_t)^2 \left[\frac{ktd}{(d + \alpha t)^2} \right]^2 \qquad (23)$$

Plane Anisotropic Case

For this case an additional variable must be considered: $r = k_y/k_x$.. Thus, basic equations become:

$$q = \frac{k_x h^2}{2d} + \frac{k_x \sqrt{r}\, ht}{d\sqrt{r} + \alpha t} \qquad \alpha = 0.44 \rightarrow op/t > 0.5 \qquad (24)$$

$$E[q] \cong \mu_{k_x} \left[\frac{\mu_h^2}{2d} + \frac{\mu_h \mu_t \sqrt{\mu_r}}{d\sqrt{\mu_r} + \alpha \mu_t} \right], \text{ with} \qquad \alpha = 0.44 \rightarrow op/t > 0.5 \qquad (25)$$

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[10] Rigorous solutions must consider slope geometry parameters such as “d” and “op”.

$$VAR[q] \cong (\sigma_{k_x})^2 \left[\frac{h^2}{2\mathrm{d}} + \frac{ht\sqrt{r}}{d\sqrt{r}+\alpha t} \right]^2 + (\sigma_h)^2 \left[\frac{k_x h}{\mathrm{d}} + \frac{t\sqrt{r}}{d\sqrt{r}+\alpha t} \right]^2 + \ldots\ldots$$

$$\ldots\ldots\ldots\ldots + (\sigma_t)^2 \left[\frac{hk_x dr}{(d\sqrt{r}+\alpha t)^2} \right]^2 + (\sigma_r)^2 \left[\frac{\frac{1}{2}\alpha hk_x t^2\sqrt{r}}{r(d\sqrt{r}+\alpha t)^2} \right]^2 \qquad (26)$$

A series of analyses were run with values of k, h and t, with mean values μ_k, μ_h and μ_t, and standard deviations σ_k, σ_h and σ_t, respectively, performing Latin hyper-cube simulations using a spreadsheet and @Risk software and compared with FOSM methodology. **Figure 9** shows the mean value and distribution of quantity of flow (Q) for the performed simulations (lines), considering a) keeping constant the slope geometry parameters (μ_1); b) considering variation of slope geometry parameters (μ_2), to show the effects of the slope geometry (rigorous solution). Both cases consider normal distributions for the other parameters. Also, this figure shows the effect of considering lognormal distribution for the hydraulic conductivity (μ_3). **Table 1** show the summary results, so it is clear that for analyzed cases, the mean value and standard deviation of the quantity of flow obtained from the FOSM method is a good approximation.

Table 1 : Summary results of analyzed cases

Case	Parameters & Distribution	Mean Value μ (m^2/s)	Standard Deviation σ
1 (Simulation)	h, k, t (Normal)	1.450×10^{-3}	6.86×10^{-4}
2 (Simulation)	All parameters (Normal)	1.454×10^{-3}	6.97×10^{-4}
3 (Simulation)	k_1 & k_2 (Lognormal) / Rest (Normal)	1.436×10^{-3}	7.24×10^{-4}
4 (FOSM)	--	1.415×10^{-3}	8.13×10^{-4}
5 (Numerical Methods)	--	1.390×10^{-3}	--

CONCLUSIONS AND FUTURE WORK

Equations (10), (11), (14), (15), (18) and (19), provide a quick method of estimating the quantity of flow for open pit slopes. These equations include the effects of slope geometry, anisotropy and non-homogeneous materials. The equations were derived using Darcy's Law (confined flow) and Dupuit's assumptions (unconfined flow) and validated with results from finite element analyses carried out over the reasonable range of parameters likely to be encountered in practice.

The results presented here are valid for the range of parameters used to derive them. In particular, two points should be borne in mind. First, for ratios op/t ≤ 0.5, especially for the anisotropic case ($k_h > k_v$), small "openings" (op) restricted the flow and consequently reduce the overall flow (impeded flow). If the horizontal hydraulic conductivity is excessively large, the transformed section will have the boundaries too close, the estimated of flow will be low.

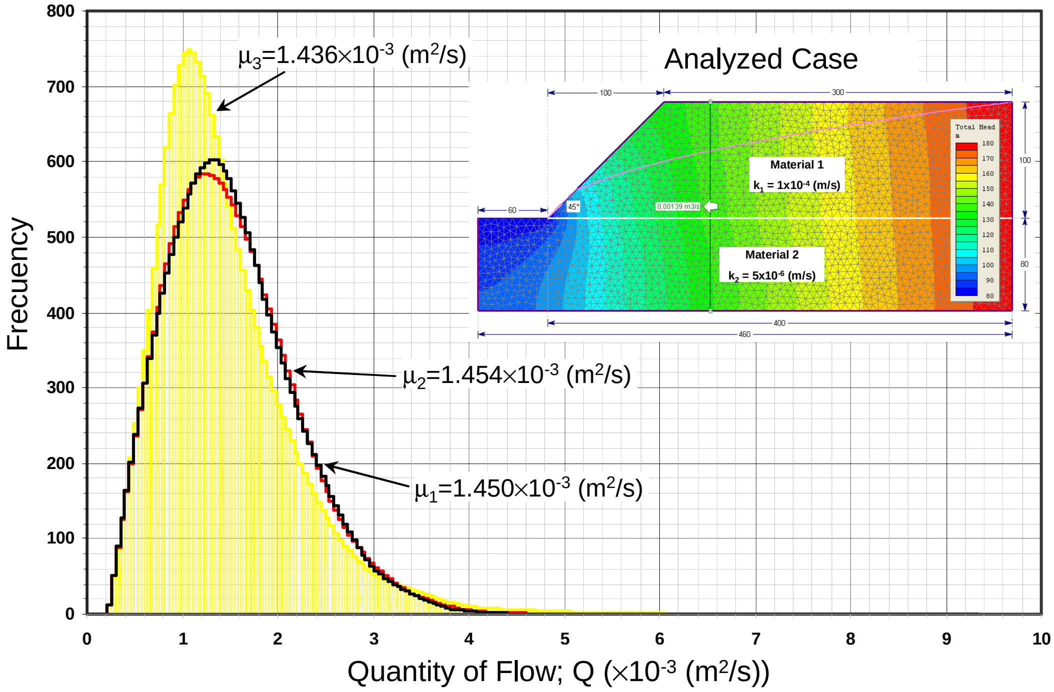

Figure 9: Mean values and distributions of quantity of flow (Q) for the performed simulations (lines), to show the effects of the slope geometry (rigorous solution). Both cases consider normal distributions for the others parameters. Also, this figure shows the effect of considering of lognormal distributions (shaded zone) for the hydraulic conductivities (μ_3). Coefficient of Variations used for h = 0.2; for k_1 and k_2 = 0.5; and for d, t and op = 0.1.

For the equations derived here, the FOSM was applied and compared with Latin hypercube simulations using @Risk software for several statistical distributions of hydraulic conductivity, total head and slope geometries. It is clear that for most cases, the mean value and standard deviation of the quantity of flow obtained from the FOSM method is a good approximation.

Further work with this topic should focus on a closer examination of the axi-symmetric solution for the isotropic and anisotropic case. Determination of the statistical distribution of quantity of flow is part of ongoing research because of the significant variability of hydraulic conductivity (at least one order of magnitude).

Acknowledgments: We would like to acknowledge the cooperative efforts from Mr. Ivan Romero and Claudio Bisso from CMDIC.

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